Distributed Monitoring and Control of Load Tap Changer Dynamics

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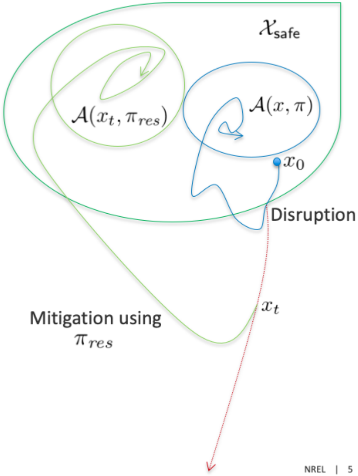
Introduction

Resilience (GMLC report)

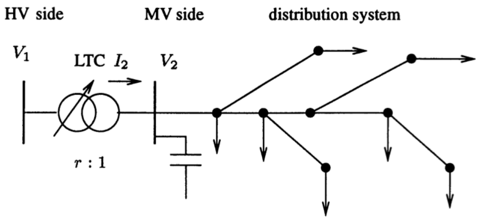
The ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions.

- Understanding the stability of networked LTCs.
  - Stable equilibrium point.
  - Region of attraction characterization.
- Designing algorithm for distributed monitoring and control.



Load Tap Changer (LTC)

Voltage regulation device that controls the voltage of the Medium Voltage (MV) side by changing the transformer ratio r.



$$r_{k+1} = \begin{cases} r_k + \Delta r & \text{if } V_2 > V_2^0 + d \text{ and } r_k < r^{\max} \\ r_k - \Delta r & \text{if } V_2 < V_2^0 - d \text{ and } r_k > r^{\min} \\ r_k & \text{otherwise} \end{cases}$$

Continuous approximation:

$$\dot{r} = \frac{1}{T_c}(V_2 - V_2^0) \qquad r^{\min} \leq r \leq r^{\max}$$

Instability Mechanism

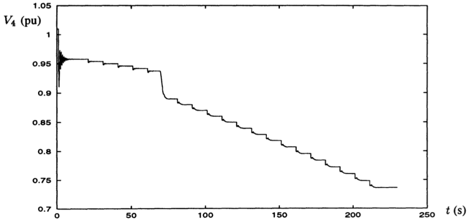


Figure 8.10 Case 1: Transmission side voltage

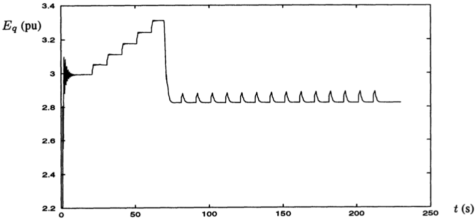


Figure 8.11 Case 1: Generator field current

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Dynamical System Model

The dynamic of networked LTC is governed by the following equation:

$$\dot{r}_i = \frac{1}{T_i}(V_{s,i}(\boldsymbol{r}) - V_{0,i}), \quad \forall i \in \mathcal{V}_L$$

where

$$\boldsymbol{V}_s = -\boldsymbol{B}_{LL}^{-1}[\boldsymbol{r}]^{-1}\boldsymbol{B}_{LG}\boldsymbol{V}_G.$$

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The set of equilibria of the dynamical system is

$$\mathcal{M} = \{\boldsymbol{r} \in \mathbb{R}^n_{>0} : \ \dot{r}_i(\boldsymbol{r}) = 0, \ \forall i \in \mathcal{V}_L\}.$$

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Define the set $\mathcal{P}$ as

$$\mathcal{P} = \left\{\boldsymbol{r} \in \mathbb{R}^n_{>0} : \ \dot{r}_i(\boldsymbol{r}) \geq 0, \ \forall i \in \mathcal{V}_L\right\}.$$

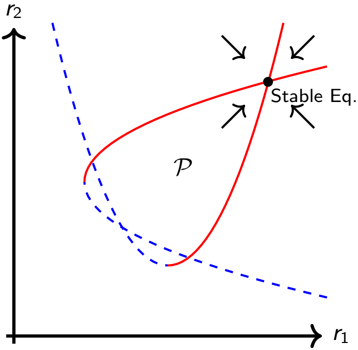
Note that $\mathcal{M}$ lies on the boundary of $\mathcal{P}$.

Stability Analysis and ROA Characterization

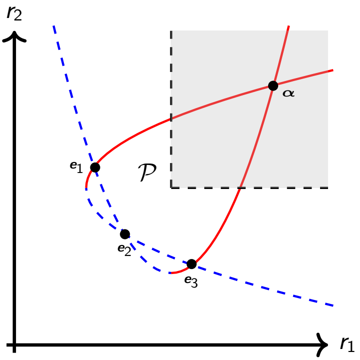
Stable Equilibrium

Equilibria are the intersection of quadratic hypersurfaces. Two things are known:

- A maximum equilibrium exists;
- It is asymptotically stable.



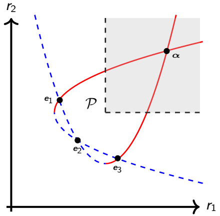
Region of Attraction



Theorem (Liu & Vu, 89')

The set $\mathcal{A}(\boldsymbol{r}^*) := \{\boldsymbol{r} : \boldsymbol{r} \geq \boldsymbol{r}^*\}$ is a region of attraction of $\boldsymbol{\alpha}$ if $\boldsymbol{r}^* \in \mathcal{P}$ and $\boldsymbol{\alpha}$ is the only equilibrium point in $\mathcal{A}(\boldsymbol{r}^*)$.

Region of Attraction



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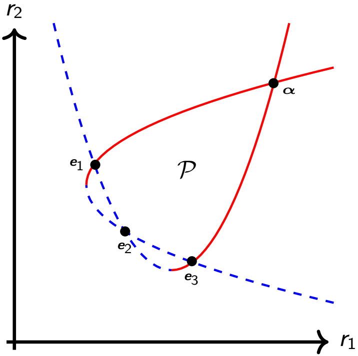
Computational considerations

- Efficient characterization of $\mathcal{P}$?
- How to ensure no other equilibria in $\mathcal{A}(\boldsymbol{r}^*)$?

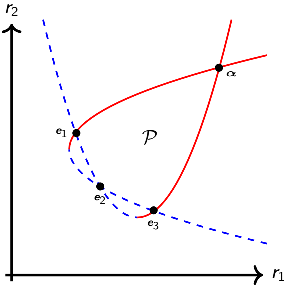
Region of Attraction



Observations from the figure

1. The set $\mathcal{A}(\boldsymbol{r})$ contains exactly one equilibrium point for all $\boldsymbol{r} \in \mathrm{int}(\mathcal{P})$.
2. All equilibria other than $\boldsymbol{\alpha}$ are unstable.
3. $\mathcal{P}$ is convex.

Main Results



Proposition

All equilibria other than $\boldsymbol{\alpha}$ are unstable.

Proposition

There is a unique equilibrium point in $\mathcal{A}(\boldsymbol{r}^*) = \{\boldsymbol{r} : \boldsymbol{r} \geq \boldsymbol{r}^*\}$ for any $\boldsymbol{r}^* \in \mathcal{P} \setminus \mathcal{M}$.

Corollary

The set $\mathcal{A}(\boldsymbol{r}^*)$ is a region of attraction of $\boldsymbol{\alpha}$ for any $\boldsymbol{r}^* \in \mathcal{P}$.

Stability Monitoring and Control

Stability Assessment

A tap position vector $\boldsymbol{r}_0$ is inside the region of attraction if there is $\boldsymbol{r}^*$, $\boldsymbol{V}^*$ such that

$$\left(\tilde{\boldsymbol{B}}_{LL} + [\boldsymbol{b}_s][\boldsymbol{r}^*]^{-2}\right)\boldsymbol{V}^* = \boldsymbol{h},\quad [\boldsymbol{r}^*]^{-1}\boldsymbol{V}^* \geq \boldsymbol{V}_0,\quad \mathbb{0} \leq \boldsymbol{r}^* \leq \boldsymbol{r}_0.$$

$$\xrightarrow{\boldsymbol{u}^* = [\boldsymbol{r}^*]^{-2}\boldsymbol{V}^*}$$

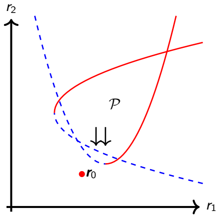
$$\tilde{\boldsymbol{B}}_{LL}\boldsymbol{V}^* + [\boldsymbol{b}_s]\boldsymbol{u}^* = \boldsymbol{h},\quad [\boldsymbol{V}^*]\boldsymbol{u}^* \geq [\boldsymbol{V}_0]\boldsymbol{V}_0,\quad [\boldsymbol{u}^*]^{-1}\boldsymbol{V}^* \leq [\boldsymbol{r}_0]\boldsymbol{r}_0,\quad \boldsymbol{V} \geq \mathbb{0}.$$

Convex optimization formulation

$$\begin{aligned} \min_{\boldsymbol{u},\boldsymbol{V}\geq\mathbb{0}} \quad & \left\|\tilde{\boldsymbol{B}}_{LL}\boldsymbol{V} + [\boldsymbol{b}_s]\boldsymbol{u} - \boldsymbol{h}\right\|^2 \\ \text{s.t.} \quad & [\boldsymbol{V}]\boldsymbol{u} \geq [\boldsymbol{V}_0]\boldsymbol{V}_0, \\ & [\boldsymbol{u}]^{-1}\boldsymbol{V} \leq [\boldsymbol{r}_0]\boldsymbol{r}_0. \end{aligned}$$

Instability Mitigation

Minimum change in load such that the tap position vector $\boldsymbol{r}_0$ is in $\mathcal{P}$:



Minimum effort to guarantee resilience

$$\min_{\boldsymbol{V},\boldsymbol{r},\boldsymbol{d}} \quad \|\boldsymbol{d}\|^2$$

$$\text{s.t.} \quad \left(\tilde{\boldsymbol{B}}_{LL} + [\boldsymbol{b}_s - \boldsymbol{d}][\boldsymbol{r}]^{-2}\right)\boldsymbol{V} = \boldsymbol{h}$$

$$[\boldsymbol{r}]^{-1}\boldsymbol{V} \geq \boldsymbol{V}_0$$

$$\mathbb{0} \leq \boldsymbol{r} \leq \boldsymbol{r}_0$$

$$\mathbb{0} \leq \boldsymbol{d} \leq \boldsymbol{b}_s.$$

- Nonconvex: only admits a convex inner approximation.

Instability Mitigation: Convex Inner Approximation

$$
\begin{aligned}
\min_{\boldsymbol{V} \geq \mathbb{0}, \boldsymbol{u}, \boldsymbol{d}} \quad & \|\boldsymbol{d}\|_2^2 \\
\text{s.t.} \quad & \tilde{\boldsymbol{B}}_{LL} \boldsymbol{V} + [\boldsymbol{b}_s]\boldsymbol{u} - [\boldsymbol{d}]\boldsymbol{u} = \boldsymbol{h} \\
& [\boldsymbol{V}]\boldsymbol{u} \geq [\boldsymbol{V}_0]\boldsymbol{V}_0, \\
& [\boldsymbol{u}]^{-1}\boldsymbol{V} \leq [\boldsymbol{r}_0]\boldsymbol{r}_0, \\
& \mathbb{0} \leq \boldsymbol{d} \leq \boldsymbol{b}_s.
\end{aligned}
$$

Instability Mitigation: Convex Inner Approximation

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& [\boldsymbol{u}]^{-1}\boldsymbol{V} \leq [\boldsymbol{r}_0]\boldsymbol{r}_0, \\
& \mathbb{0} \leq \boldsymbol{d} \leq [\boldsymbol{b}_s]\boldsymbol{u}
\end{aligned}
$$

Stability Monitoring and Control

- Both monitoring and mitigation problems can be formulated as a single problem:

$$
\begin{aligned}
\min_{\boldsymbol{u}, \boldsymbol{V} \geq \mathbb{0}} \quad & \left\| \tilde{\boldsymbol{B}}_{LL} \boldsymbol{V} + [\boldsymbol{b}_s] \boldsymbol{u} - \boldsymbol{h} \right\|^2 \\
\text{s.t.} \quad & \tilde{\boldsymbol{B}}_{LL} \boldsymbol{V} \leq \boldsymbol{h} \\
& [\boldsymbol{V}] \boldsymbol{u} \geq [\boldsymbol{V}_0] \boldsymbol{V}_0, \\
& [\boldsymbol{u}]^{-1} \boldsymbol{V} \leq [\boldsymbol{r}_0] \boldsymbol{r}_0.
\end{aligned}
$$

- Stable if optimal cost $= 0$.
- Can recover needed Q support otherwise.

ADMM-based Distributed Implementation

$$\min_{\boldsymbol{x},\boldsymbol{z}} \quad \sum_{i=1}^{n_s} f_i(\boldsymbol{x}^i)$$

$$\text{s.t.} \quad \boldsymbol{x}^i \in \mathcal{X}_i, \qquad i = 1, \ldots, n_s$$

$$W^{ij} = z^j, V_j = z^j, \qquad i = 1, \ldots, n_s,\ j \in \mathcal{N}_i^a$$

- n_s: number of connected subgraphs (agents)
- $\mathcal{N}_i$: the bus set of the ith agent;
- $\mathcal{N}_i^a$: the set of buses adjacent to the ith agent;
- $\boldsymbol{x}^i = \left(\{V_j\}_{j\in\mathcal{N}_i}, \{u_j\}_{j\in\mathcal{N}_i}, \{W^{ij}\}_{j\in\mathcal{N}_i^a}\right)$ collect the optimization variables of agent i

The corresponding augmented Lagrangian with penalty parameter ρ is

$$L_\rho(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i=1}^{n_s} f_i(\boldsymbol{x}^i) + \sum_{i\in\mathcal{B}} \left(\lambda^i(V_i - z^i) + \frac{\rho}{2}(V_i - z^i)^2\right)$$
$$+ \sum_{i=1}^{n_s} \sum_{j\in\mathcal{N}_i^a} \left(\mu^{ij}(W^{ij} - z^j) + \frac{\rho}{2}(W^{ij} - z^j)^2\right).$$

ADMM-based Distributed Implementation

ADMM performs the following iterative updates:

$$\begin{aligned}
\boldsymbol{x}_{k+1}^i = \arg\min_{\boldsymbol{x}^i \in \mathcal{X}_i} \Bigg\{ f_i(\boldsymbol{x}^i) + \sum_{j \in \mathcal{B} \cup \mathcal{N}_i} \left(\lambda_k^j V_j + \frac{\rho}{2}(V_j - z_k^j)^2 \right) \\
+ \sum_{j \in \mathcal{N}_i^a} \left(\mu_k^{ij} W^{ij} + \frac{\rho}{2}(W^{ij} - z_k^j)^2 \right) \Bigg\}, \qquad \forall i \in [n_s]
\end{aligned}$$

$$z_{k+1}^i = \arg\min \left\{ \sum_{j: i \in \mathcal{N}_j^a} \left(-\mu_k^{ji} z^i + \frac{\rho}{2}(W_{k+1}^{ji} - z^i)^2 \right) - \lambda_k^i z^i + \frac{\rho}{2}(V_{i,k+1} - z^i)^2 \right\},$$

$$\lambda_{k+1}^i = \lambda_k^i + \rho \left(V_{i,k+1} - z_{k+1}^i \right), \qquad \forall i \in \mathcal{B}$$

$$\mu_{k+1}^{ij} = \mu_k^{ij} + \rho \left(W_{k+1}^{ij} - z_{k+1}^j \right), \qquad \forall i, \forall j \in \mathcal{N}_i^a$$

ADMM-based Distributed Implementation

Initialization: *Multipliers* $\boldsymbol{\mu}^{(0)}, \boldsymbol{\lambda}^{(0)}$.

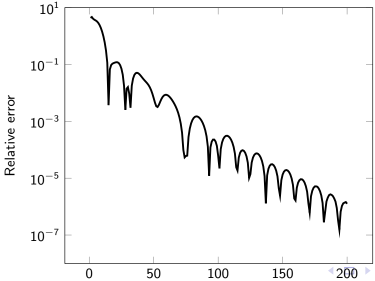
repeat

[S1] Each agent *i* receives the multipliers and voltage estimates from its neighbors, update local variables, and broadcasts the resulting voltage to its neighbors.

[S2] Each agent *i* uses its updated voltages, multipliers, and received bus voltages and multipliers to compute its voltage estimates and broadcasts them to its neighbors.

[S3] Each agent *i* updates its multipliers using its own updated bus voltages, estimated voltages, as well as received voltage estimates.

until *Primal & dual residuals are small*



Explicit ROA Characterization

ROA Characterization

Finding minimum tap position in $\mathcal{P}$ along certain direction:

$$\begin{aligned} \min_{\boldsymbol{V},\boldsymbol{r}} \quad & \boldsymbol{c}^\top \boldsymbol{r} \\ \text{s.t.} \quad & \left(\tilde{\boldsymbol{B}}_{LL} + [\boldsymbol{b}_s][\boldsymbol{r}]^{-2}\right) \boldsymbol{V} = \boldsymbol{h} && \text{(Flow constraint)} \\ & \boldsymbol{V} \geq [\boldsymbol{r}]\boldsymbol{V}_0 && \text{(Secondary side voltage requirement)} \\ & \boldsymbol{r} \geq \mathbb{0}. \end{aligned}$$

Each direction $\boldsymbol{c}$ determines a (possibly distinct) inner approximation $\mathcal{A}(\boldsymbol{r}^*(\boldsymbol{c}))$ of the true ROA, and their union characterizes a maximal inner approximation of the ROA:

$$\mathcal{A}_\cup := \bigcup_{\substack{\boldsymbol{c}\geq \mathbb{0} \\ \boldsymbol{c}^\top \mathbb{1}=1}} \mathcal{A}(\boldsymbol{r}^*(\boldsymbol{c})).$$

ROA Characterization

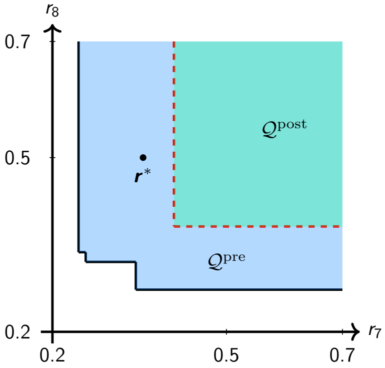


Figure: ROA Characterizations for IEEE 39-bus system before and after line tripping.

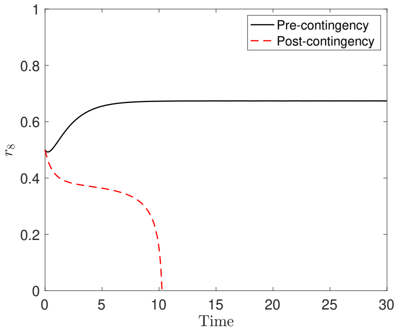


Figure: LTC dynamics at bus 8 before and after line tripping.

Ellipsoidal Inner Approximation of the ROA

$$\left(\tilde{\boldsymbol{B}}_{LL} + [\boldsymbol{b}_s][\boldsymbol{r}]^{-2}\right) \boldsymbol{V} = \boldsymbol{h}, \quad [\boldsymbol{r}]^{-1}\boldsymbol{V} \geq \boldsymbol{V}_0. \xrightarrow{\boldsymbol{u}=[\boldsymbol{r}]^{-2}V} \tilde{\boldsymbol{B}}_{LL}\boldsymbol{V} + [\boldsymbol{b}_s]\boldsymbol{u} = \boldsymbol{h}, \quad [\boldsymbol{V}]\boldsymbol{u} \geq [\boldsymbol{V}_0]\boldsymbol{V}_0.$$

The problem of finding the maximum volume inscribed ellipsoid in $\boldsymbol{V}$-space can be cast as

$$\begin{aligned} \max_{\boldsymbol{C}\succeq 0, \boldsymbol{\alpha}} \quad & \log\det \boldsymbol{C} \\ \text{s.t.} \quad & [\boldsymbol{C}\boldsymbol{\xi} + \boldsymbol{\alpha}]\boldsymbol{u}(\boldsymbol{\xi}) \geq [\boldsymbol{V}_0]\boldsymbol{V}_0 && \forall \|\boldsymbol{\xi}\|_2 \leq 1 \\ & \boldsymbol{u}(\boldsymbol{\xi}) = [\boldsymbol{b}_s]^{-1}\left(\boldsymbol{h} - \tilde{\boldsymbol{B}}_{LL}(\boldsymbol{C}\boldsymbol{\xi} + \boldsymbol{\alpha})\right) && \forall \|\boldsymbol{\xi}\|_2 \leq 1 \end{aligned}$$

Quite surprisingly, each of the robust SOC constraint above can be reformulated as an SDP with LMIs of dimension $2(n-1) \times 2(n-1)$ so the above problem admits a tractable reformulation.

Ellipoidal Inner Approximation of the ROA

Ellipsoid in $\boldsymbol{V}$-space:

$$\{\boldsymbol{V} = \boldsymbol{C}\boldsymbol{\xi} + \boldsymbol{\alpha},\ \|\boldsymbol{\xi}\|_2 \le 1\}.$$

With $\tilde{\boldsymbol{B}}_{LL}\boldsymbol{V} + [\boldsymbol{b}_s]\boldsymbol{u} = \boldsymbol{h}$, ellipsoid in $\boldsymbol{u}$-space is

$$\{\boldsymbol{u} = \boldsymbol{D}\boldsymbol{\xi} + \boldsymbol{\beta},\ \|\boldsymbol{\xi}\|_2 \le 1\}.$$

Then the Hadamard division of vectors in the two ellipsoids parametrized by the same $\boldsymbol{\xi}$ gives rise to a subset of $\mathcal{P}^2 := \{\boldsymbol{r} :\ \sqrt{\boldsymbol{r}} \in \mathcal{P}\}$ that is linear-fractional, which is

$$\mathcal{C} := \left\{\tilde{\boldsymbol{r}} \in \mathbb{R}^n_{>0} :\ \tilde{r}_i = (\boldsymbol{c}_i^\top \boldsymbol{\xi} + \alpha_i)/(\boldsymbol{d}_i^\top \boldsymbol{\xi} + \beta_i), \|\boldsymbol{\xi}\|_2 \le 1,\ \forall i \in \mathcal{V}_L\right\}.$$

We can rewrite $\mathcal{C}$ as an SOC set by introducing new variables: let $\boldsymbol{y}_i = \boldsymbol{\xi}/(\boldsymbol{d}_i^\top \boldsymbol{\xi} + \beta_i)$ and $t_i = 1/(\boldsymbol{d}_i^\top \boldsymbol{\xi} + \beta_i)$, then $\mathcal{C}$ can be rewritten as

$$\boxed{\mathcal{C} = \left\{\tilde{r} \in \mathbb{R}^n_{>0} :\ \tilde{r}_i = \boldsymbol{c}_i^\top \boldsymbol{y}_i + \alpha_i t_i, \boldsymbol{d}_i^\top \boldsymbol{y}_i + \beta_i t_i = 1, \|\boldsymbol{y}_i\|_2 \le t_i, \forall i \in \mathcal{V}_L\right\}}.$$

Simulation Results

Simulation Results: Test System

IEEE 39-bus system (decoupled Q-V model).

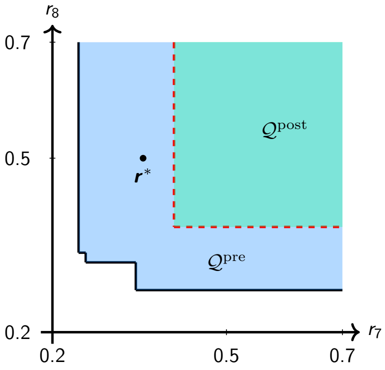


Figure: ROA Characterizations for IEEE 39-bus system before and after line tripping.

Four scenarios:

1. Steady-state (tap position in $\mathcal{Q}^{\text{post}}$) after line $(8, 9)$ outage;
2. Tap position $\boldsymbol{r}^*$ after line $(8, 9)$ outage;
3. Tap position $\boldsymbol{r}^*$ after line $(8, 9)$ outage with additional load;
4. Tap position $\boldsymbol{r}^*$ after line $(3, 4)$ outage with additional load.

Simulation Results: Convergence Rate

- Partition 39-bus system into three subsystems.
- Stopping criterion: relative error less than 10^{-4}.

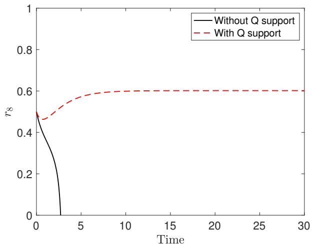


Figure: Dynamics at bus 8 (scenario 3).

Scenario	Optimal objective	# of iterations	Time (sec.)	Time per subsystem (sec.)
1	0	39	33.01	11.00
2	4.1870	89	73.52	24.51
3	12.3824	83	65.35	21.78
4	20.4829	113	109.72	36.57

Table: Simulation Results on Convergence Rate

Simulation Results: Reactive Power Support

Scenario	Total Load	Total support	Percentage
1	55.10	0	0%
2	55.10	1.93	3.50%
3	58.004	3.31	5.70%
4	58.004	4.68	8.07%

Table: Needed Reactive Power Support.

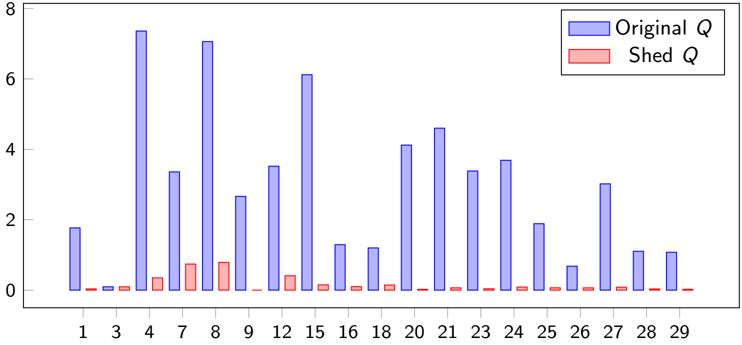


Figure: Reactive power load vs support (scenario 3).

Extension to Full Power Flow Model

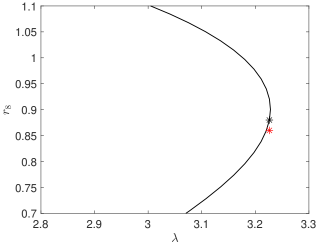


Figure: PV curve at bus 8.

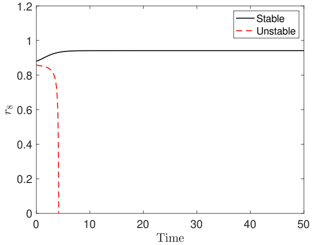
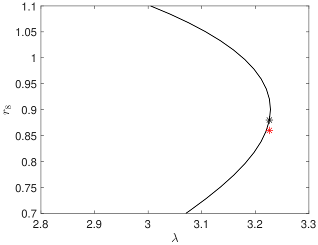


Figure: Dynamics at bus 8.

Extension to Full Power Flow Model



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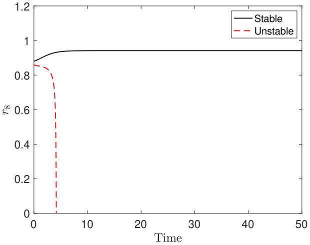

Figure: Dynamics at bus 8.

Stability Monitoring and Control Problem

$$
\begin{aligned}
\min \quad & \|\boldsymbol{Q}\|^2 & \\
\text{s.t.} \quad & P_i(c_{ii}, c_{ij}, s_{ij}) = -u_{s,i} g_{s,i} & \forall i \in \mathcal{N}, (i,j) \in \mathcal{E} \\
& Q_i(c_{ii}, c_{ij}, s_{ij}) = -u_{s,i} b_{s,i} + Q_i & \forall i \in \mathcal{N}, (i,j) \in \mathcal{E} \\
& c_{ij}^2 + s_{ij}^2 \leq c_{ii} c_{jj}, & \forall (i,j) \in \mathcal{E} \\
& u_i \geq V_{0,i}^2, \; r_{o,i}^2 u_i \geq c_{ii}, & \forall i \in \mathcal{N}.
\end{aligned}
$$

Conclusions

Summary

- New result on stability and ROA characterization of LTC dynamics
- Optimization formulations for inner approximation of ROA
- Convex formulations for stability monitoring and instability mitigation of LTC dynamics
- ADMM-based distributed implementation

Ongoing/future work

- Extension to full power flow model
- Deal with system uncertainties
- Real-time implementation

Thank you!
Questions?