On active constraints in optimal power flow

Learning optimal solutions and identifying important constraints

Line A. Roald

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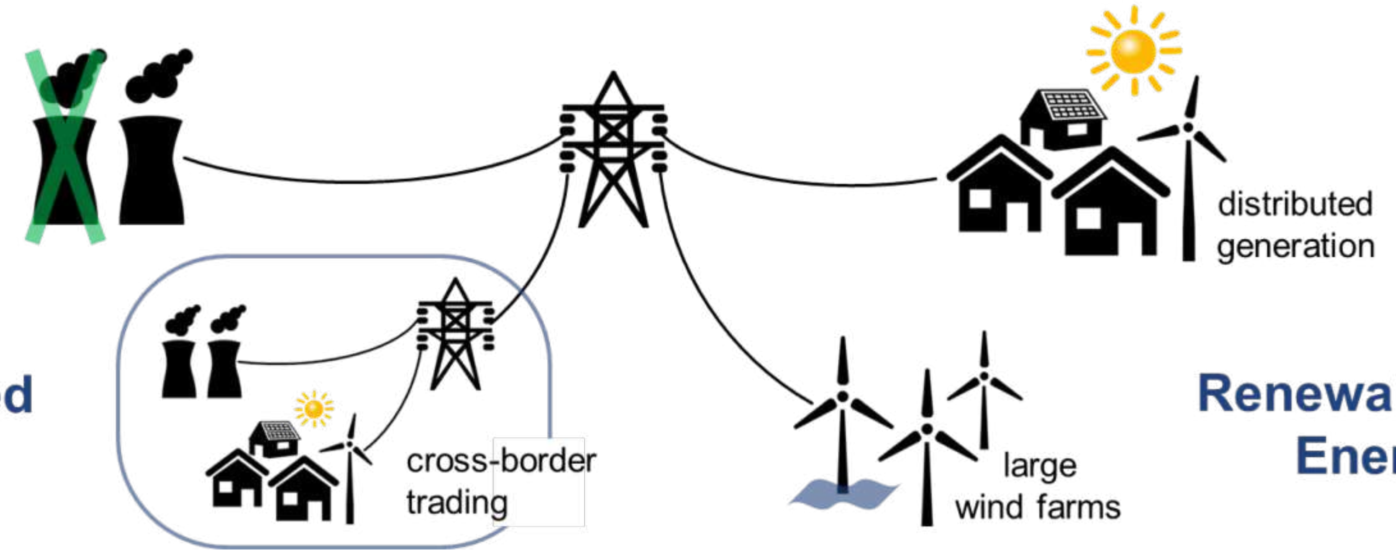
and **Daniel K. Molzahn** (Georgia Tech)

NREL Workshop, April 12 2019

Transmission System Operation

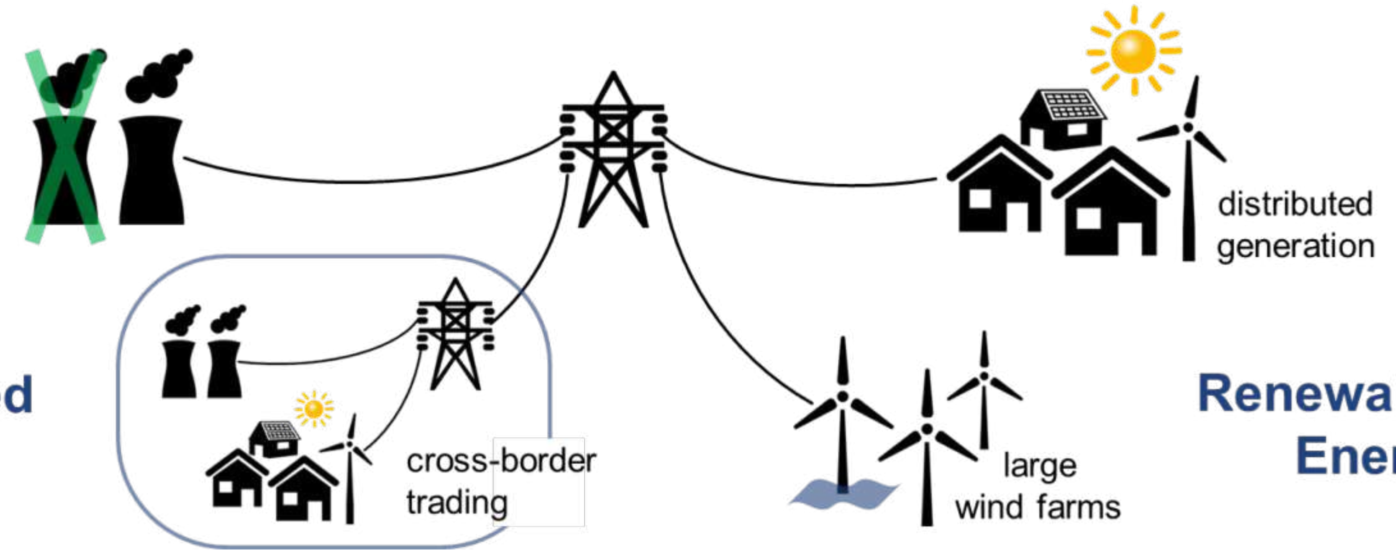
Liberalized Markets



Renewable Energy

Transmission System Operation

Liberalized Markets

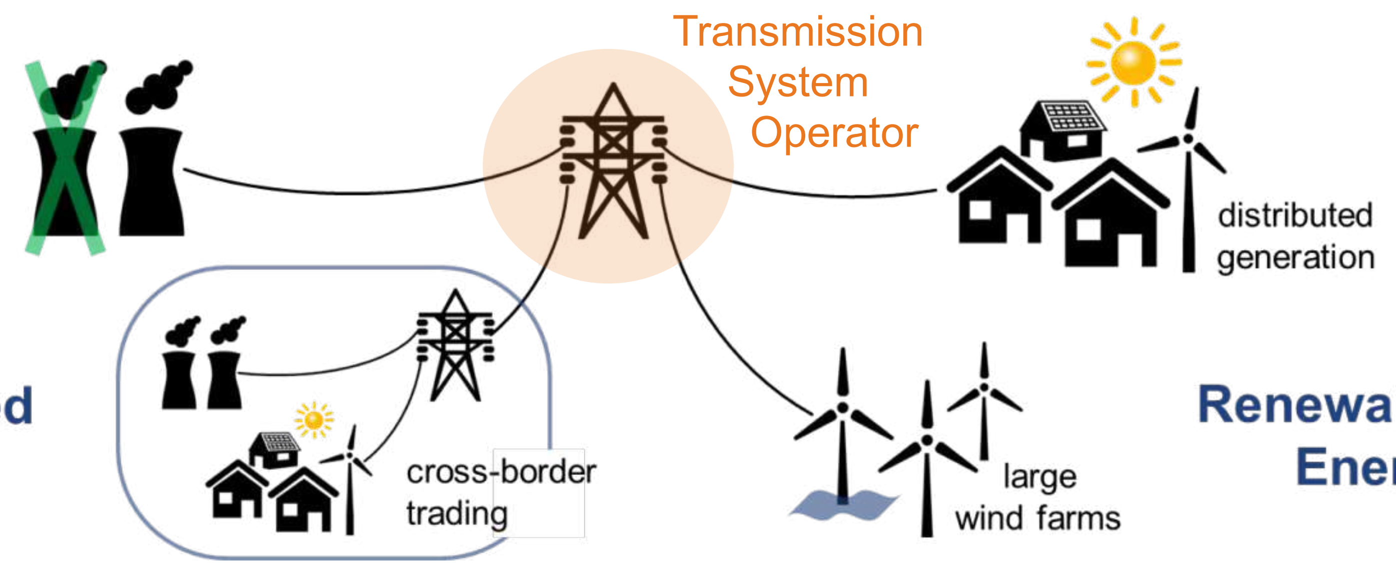


Renewable Energy

➡ Higher and frequently changing power flows

➡ Increased uncertainty

Transmission System Operation



Liberalized Markets

Renewable Energy

➡ Higher and frequently changing power flows

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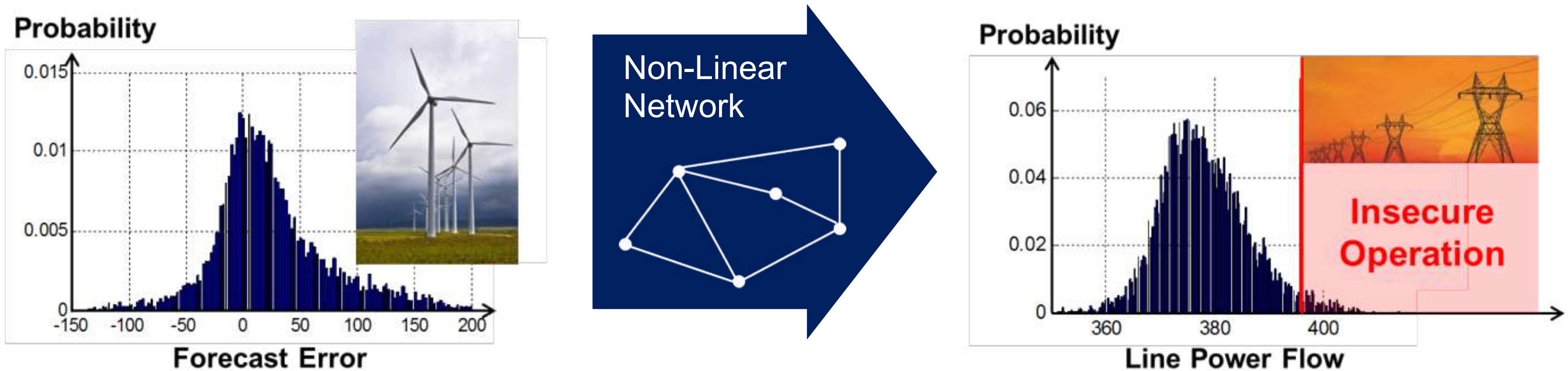
Impact of uncertainty?



How to maintain grid security?

Chance-constrained, robust, stochastic optimization

Adapt to uncertainty in real time!

The Optimal Power Flow Problem

Optimal Power Flow

Goal: Low cost operation, while enforcing technical limits

$$\min \quad \sum_{i\in\mathcal{G}}\left(c_{2,i}p_{G,i}^2 + c_{1,i}\,p_{G,i}\right)$$

Minimize generation cost

s.t.

$$f(\theta, v, p, q) = 0,$$

Non-Linear AC Power Flow

$$p_{G,g}^{min} \le p_{G,g} \le p_{G,g}^{max}, \;\; g \in \mathcal{G}$$
$$q_{G,g}^{min} \le q_{G,g} \le q_{G,g}^{max}, \;\; g \in \mathcal{G}$$

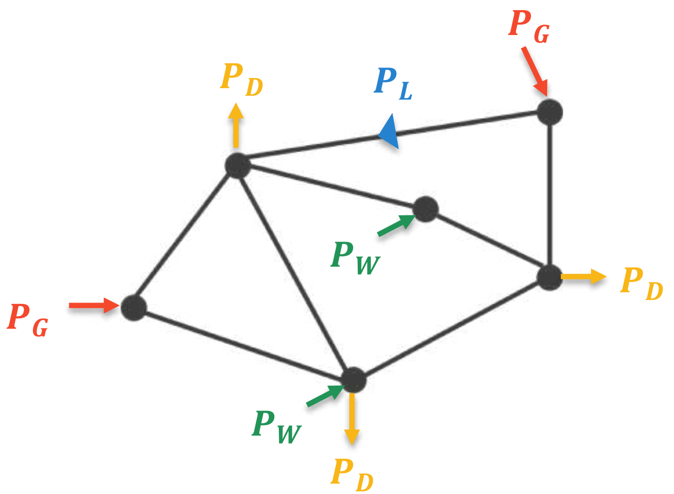
Generation constraints

$$v_i^{min} \le v_i \le v_i^{max}, \quad i \in \mathcal{B}$$

Voltage constraints

$$s_{L,j}\,(\theta, v, p, q) \le s_{L,j}^{max}, \;\; j \in \mathcal{L}$$

Transmission constraints



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Voltage constraints

$$s_{L,j}\,(\theta, v, p, q) \le s_{L,j}^{max}, \quad j \in \mathcal{L}$$

Transmission constraints

Observation 1:
Typically only *very few transmission constraints* are **active** at optimum!

Can be exploited algorithmically!

E.g. constraint generation
[Bienstock, Harnett and Chertkov, SIAM Review, 2013]
…

Optimal Power Flow

Impact of renewable energy variations/load uncertainty ω ?

$$\min \quad \sum_{i\in\mathcal{G}} \left(c_{2,i} p_{G,i}(\omega)^2 + c_{1,i}\, p_{G,i}(\omega) \right)$$ **Minimize generation cost**

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$ **Non-Linear AC Power Flow**

$$p_{G,g}^{min} \le p_{G,g}(\omega) \le p_{G,g}^{max}, \quad g \in \mathcal{G}$$
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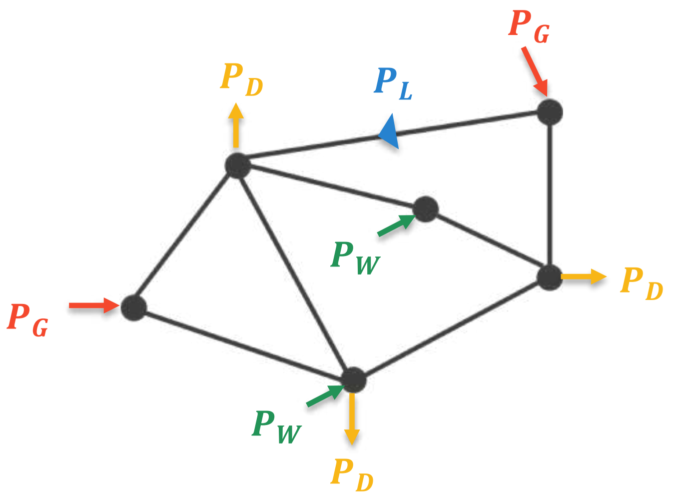
$$v_i^{min} \le v_i(\omega) \le v_i^{max}, \quad i \in \mathcal{B}$$ **Voltage constraints**

$$i_{L,j}(\omega) \le i_{L,j}^{max}, \quad j \in \mathcal{L}$$ **Transmission constraints**



Optimal Power Flow

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$$v_i^{min} \le v_i(\omega) \le v_i^{max}, \quad i \in \mathcal{B}$$ Voltage constraints

$$i_{L,j}(\omega) \le i_{L,j}^{max}, \quad j \in \mathcal{L}$$ **Transmission constraints**

Observation 2:
Typically only *very few transmission constraints* are **ever** active even for different parameters ω !

The topic of this talk!

Is this observation true??

How can we use it??

1. **Learning solutions to (power system) optimization problems through optimal active sets**

2. **Identifying potentially active constraints**

Learning solutions to (power system) optimization problems

Sidhant Misra
(LANL)

Yee Sian Ng
(MIT)

Yee Sian Ng, Sidhant Misra, Line Roald and Scott Backhaus, «Statistical Learning for DC Optimal Power Flow», Power System Computation Conference (PSCC), 2018

Sidhant Misra, Line Roald and Yee Sian Ng, «Learning for Constrained Optimization», submitted, available online: https://arxiv.org/abs/1802.09639

Optimal Power Flow

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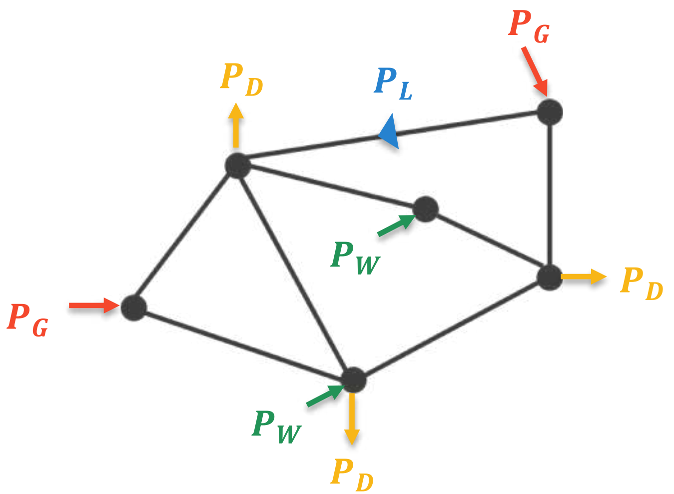

Optimal Power Flow

Resolve problem every 5-15 min! For each ω, obtain $p_G^*(\omega)$

$$\min \quad \sum_{i\in\mathcal{G}}\left(c_{2,i}p_{G,i}(\omega)^2 + c_{1,i}\,p_{G,i}(\omega)\right)$$

Minimize generation cost

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

Non-Linear AC Power Flow

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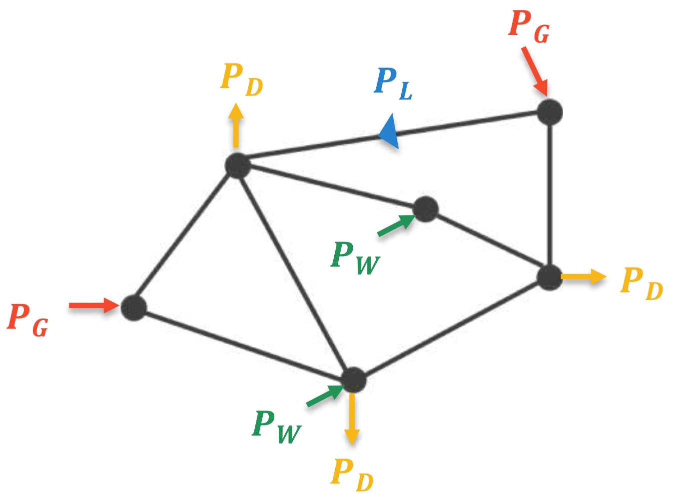
Transmission constraints



Repeated solution process

OPF at T_1: $\omega_1 \rightarrow p_G^*(\omega_1)$

$$\min_{p_G(\omega)} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}(\omega)^2 + c_{1,i}\, p_{G,i}(\omega) \right)$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

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$$v_i^{min} \leq v_i(\omega) \leq v_i^{max}, \quad i \in \mathcal{B}$$

$$i_{L,j}(\omega) \leq i_{L,j}^{max}, \quad j \in \mathcal{L}$$

OPF at T_2: $\omega_2 \rightarrow p_G^*(\omega_2)$

$$\min_{p_G(\omega)} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}(\omega)^2 + c_{1,i}\, p_{G,i}(\omega) \right)$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

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OPF at T_3: $\omega_3 \rightarrow p_G^*(\omega_3)$

$$\min_{p_G(\omega)} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}(\omega)^2 + c_{1,i}\, p_{G,i}(\omega) \right)$$

s.t.

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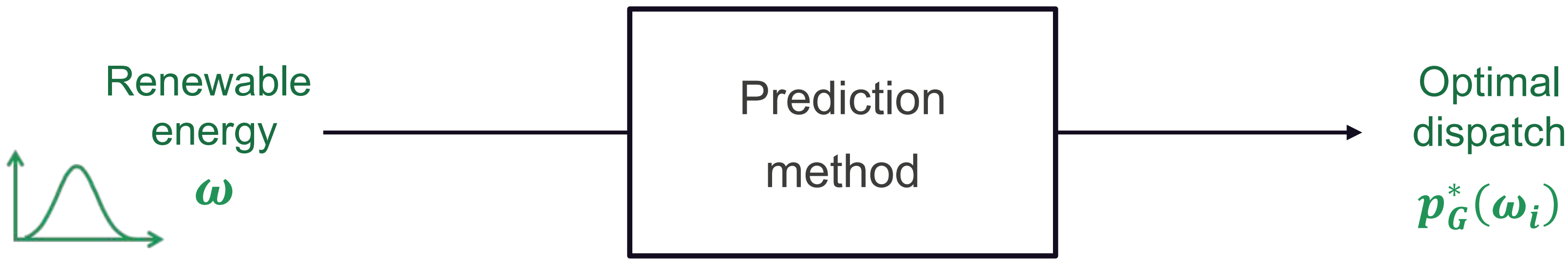
…

Can we use learning to speed up the solution process

by using information from past solutions $\left(\omega_i, p_G^*(\omega_i)\right)$?

Learning for optimization

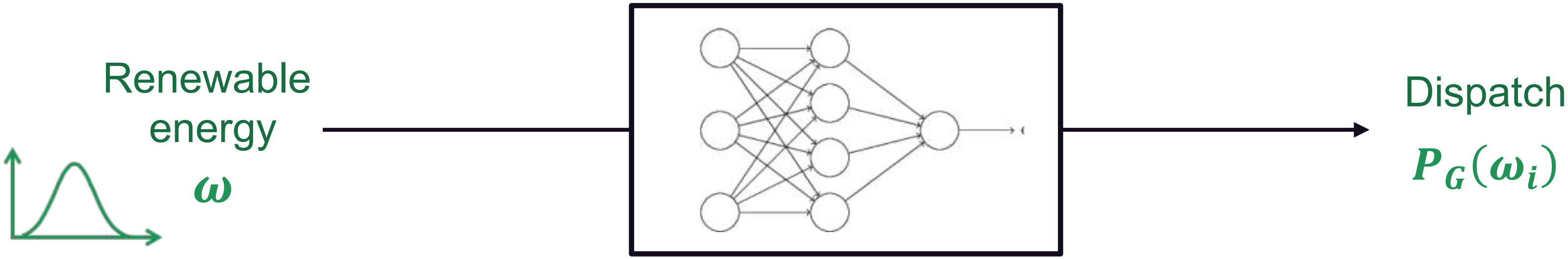


Can we use learning to speed up the solution process by using information from past solutions $\left(\omega_i, p_G^*(\omega_i)\right)$ **?**

First attempt:

Train a neural net

First Attempt – Train a Neural Net



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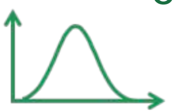
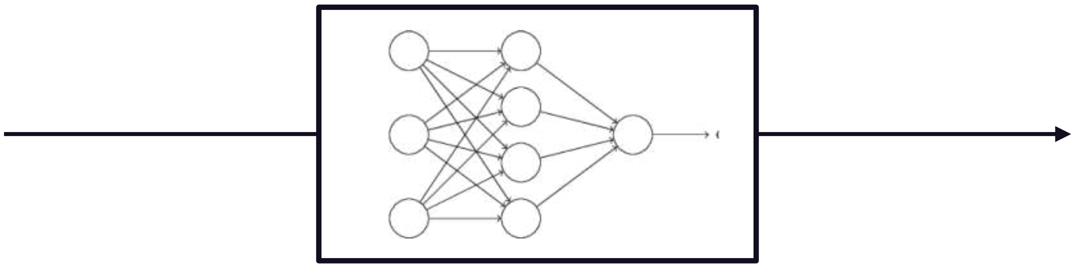


Renewable energy ω



Dispatch $P_G(\omega_i)$

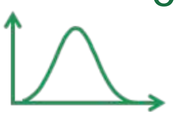
- **This didn't work well...**

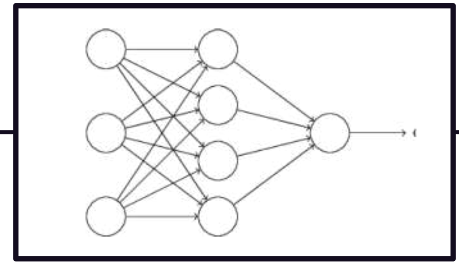
(DISCLAIMER: I will admit that we gave up quite fast!)

First Attempt – Train a Neural Net

Renewable energy

ω





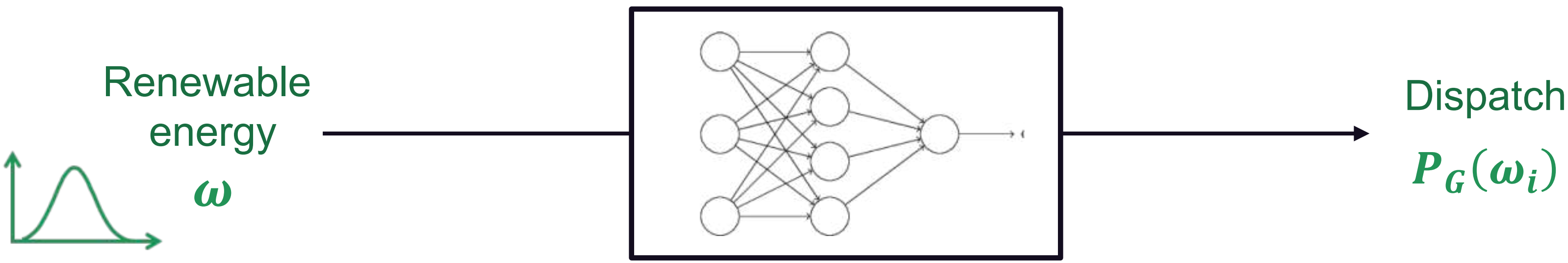
Dispatch

$P_G(\omega_i)$

- **This didn’t work well...**
  - – Hard to satisfy safety constraints!

In-depth literature review: Sidhant Misra, Line Roald and Yee Sian Ng, «Learning for Constrained Optimization», submitted, available online: https://arxiv.org/abs/1802.09639

First Attempt – Train a Neural Net



- **This didn't work well...**
  - Hard to satisfy safety constraints!
  - Projection back onto feasible space cause suboptimality…

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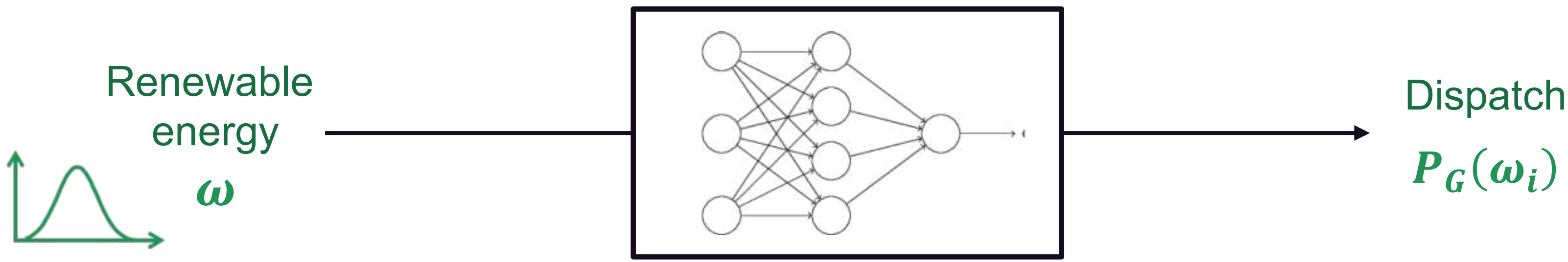

- **This didn't work well...**
  - – Hard to satisfy safety constraints!
  - – Projection back onto feasible space cause suboptimality…
  - – Challenging: High-dimensional input → *High dimensional output*

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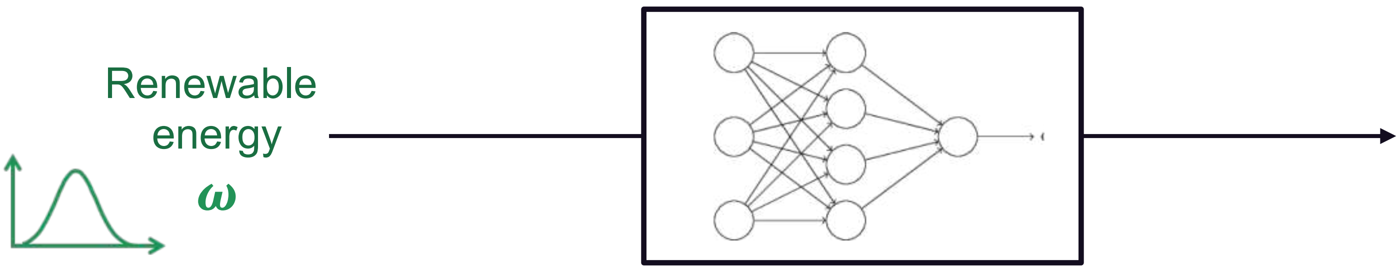
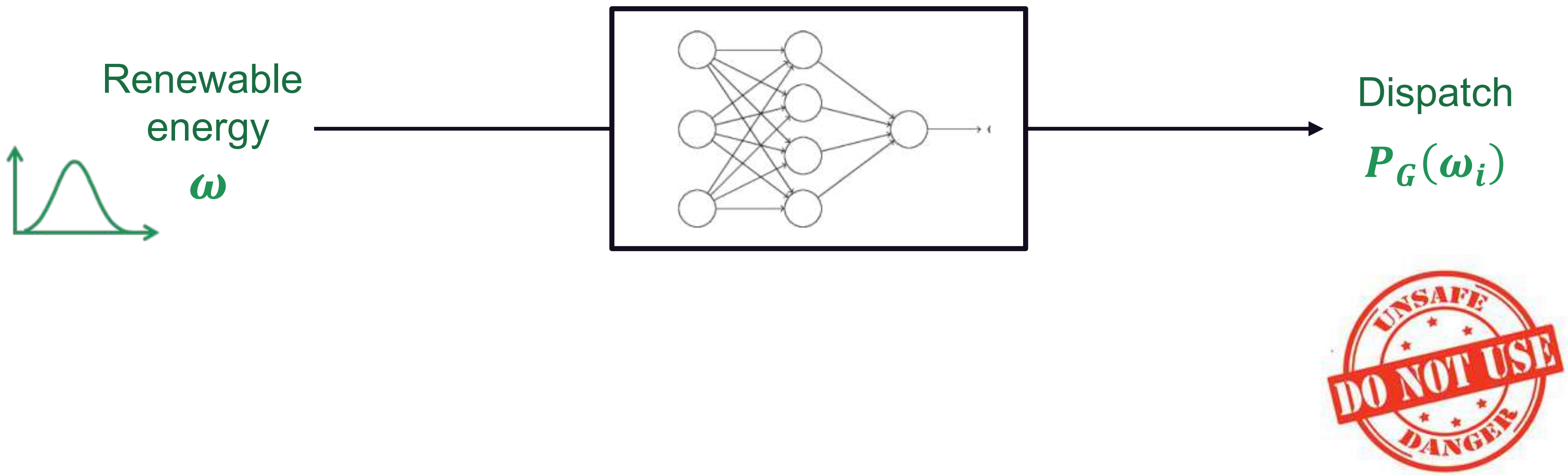
First Attempt – Train a Neural Net



- **This didn't work well...**
  - Hard to satisfy safety constraints!
  - Projection back onto feasible space cause suboptimality…
  - Challenging: High-dimensional input → *High dimensional output*
- **This can work well under some circumstances**

Wide enough and deep enough, and with enough data! [Karg and Lucia, 2018]

First Attempt – Train a Neural Net



We have a **mathematical optimization problem**

– can we use **more information** about the **problem structure**?

Think again:

How can we leverage pre-existing knowledge about the solution?

Think again:

How can we leverage pre-existing knowledge about the solution?

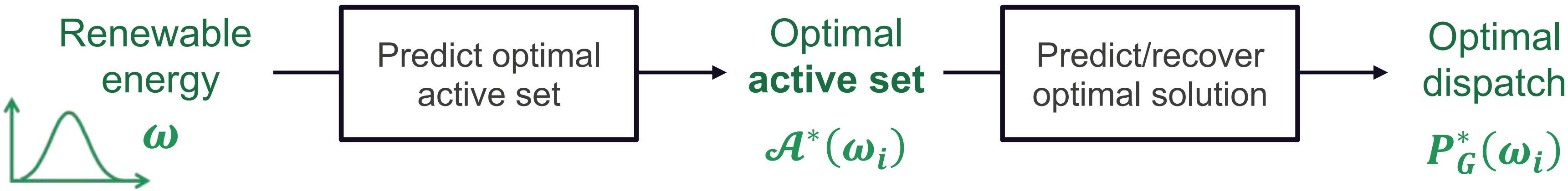
New idea:
Learn the optimal set of active constraints!

Optimal set of active constraints

Set of constraints that are active at optimum!

- Equality (power flow) constraints are **always** active
- Only **very few** of the inequality constraints are active
  - Generation constraints
  - Voltage constraints
  - Transmission constraints

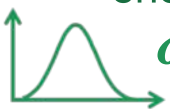
Learn optimal set of active constraints



- **Why?**
  - Optimal active set is the “minimal” information we need to recover optimal solution
  - Inherently encodes information about physical constraints and technical limits
  - Finite, low dimensional object
  - Nice physical interpretation (power system operational pattern)

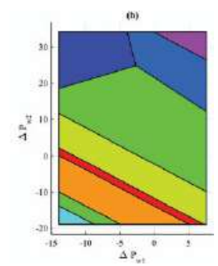
Learn optimal set of active constraints

Renewable energy ω



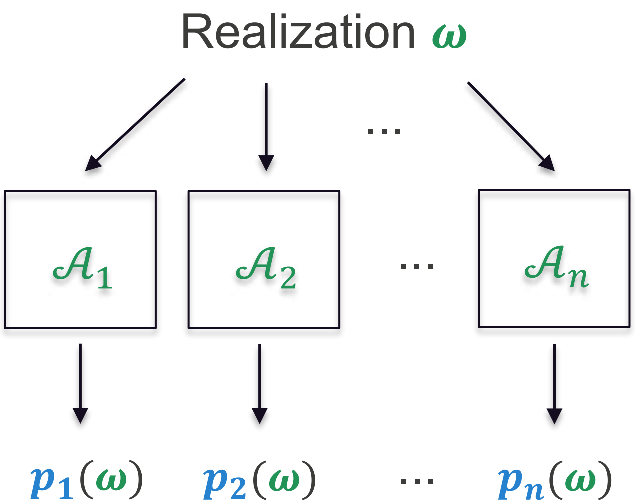
→ Predict optimal active set → Optimal **active set** $\mathcal{A}^*(\omega_i)$ → Predict/recover optimal solution → Optimal dispatch $P_G^*(\omega_i)$

- **Related to explicit MPC**
  - Explicit MPC – each optimal active set corresponds to an optimal affine control policy
    [Bemporad et al, 2002], [Pannochia, Rawlings, Wright, 2007], [Zeilinger et al, 2011], [Karg and Lucia, 2018], …



Look only at specific classes of problems
Not very scalable
Do not consider input distribution over ω

Ensemble Policy



Candidates for optimal active set

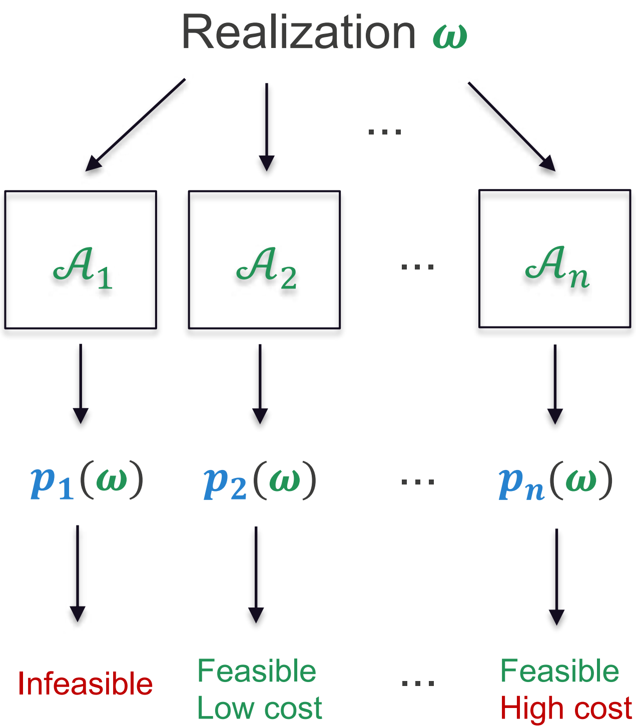
Solve problem given the active set

→ Solve a reduced problem with fewer constraints!

→ Solve a set of linear equations (linear problem)!

Easier than solving the full optimal power flow problem

Ensemble Policy

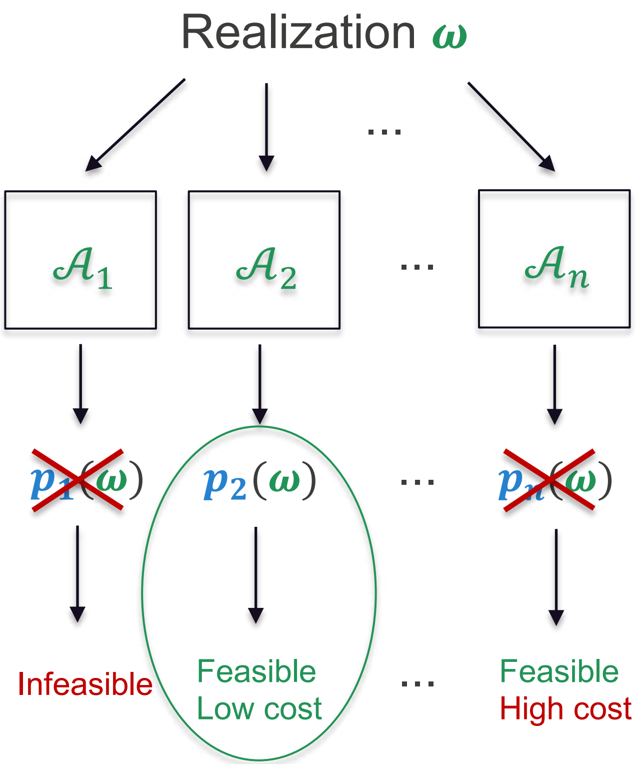


Candidates for optimal active set

Solve problem given the optimal active set

Evaluate cost and feasibility

Ensemble Policy



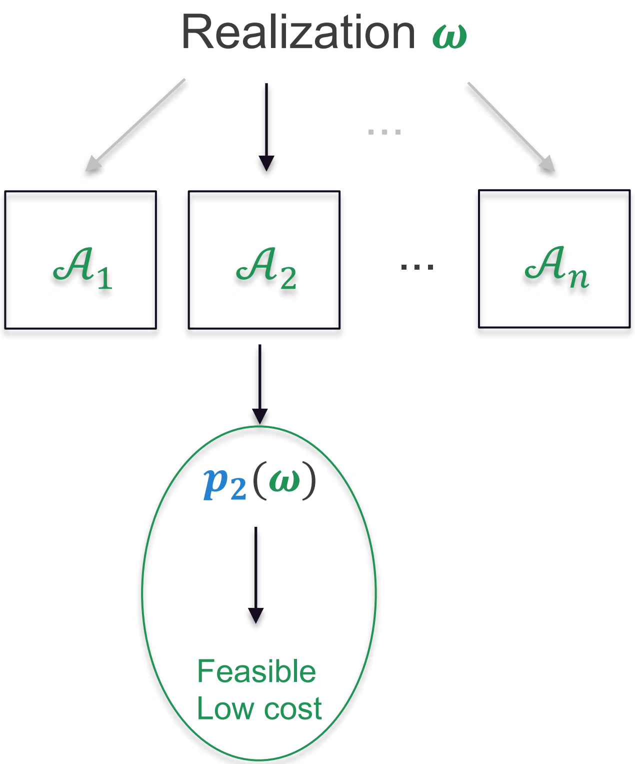
Candidates for optimal active set

Solve problem given the optimal active set

Evaluate cost and feasibility

Pick **best (optimal?)** solution

Limits of the Approach

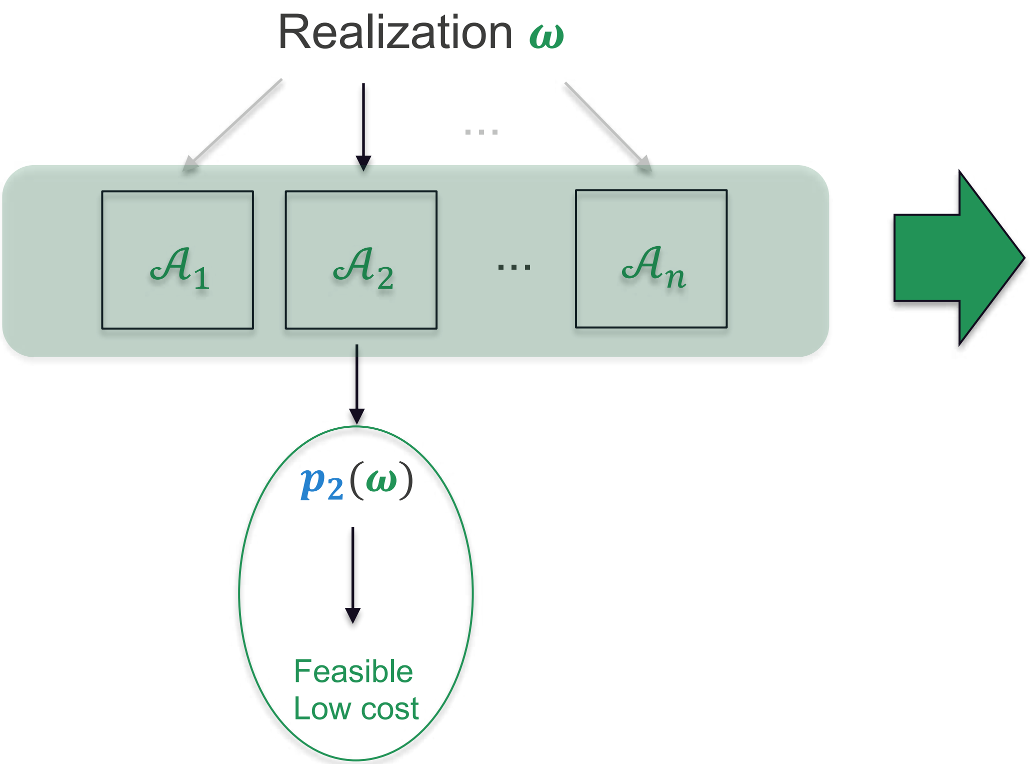


Works well if the number of active sets $\mathcal{A}_n$ is **small**!

Total number of possible active sets is exponential ☹

Maybe only a few are practically relevant? ☺

Limits of the Approach



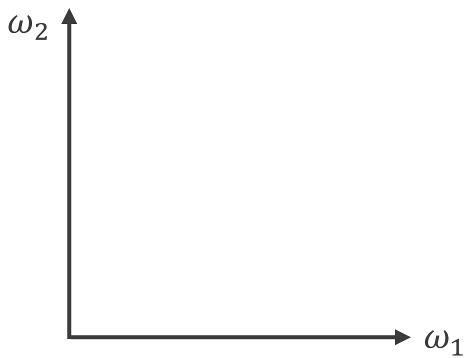
How to identify the collection of **relevant** active sets?

High probability that one of the active sets is optimal for a new realization ω

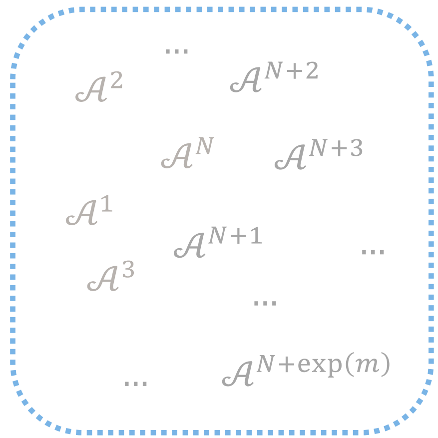
Do NOT search entire parameter space!

Using Sampling to Learn Important Active Sets

Learning Collection of Optimal Active Sets



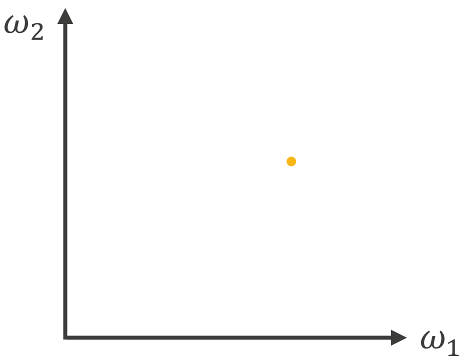
- samples of input parameters



all possible active sets

color: discovered active sets
grey: undiscovered active sets

Learning Collection of Optimal Active Sets



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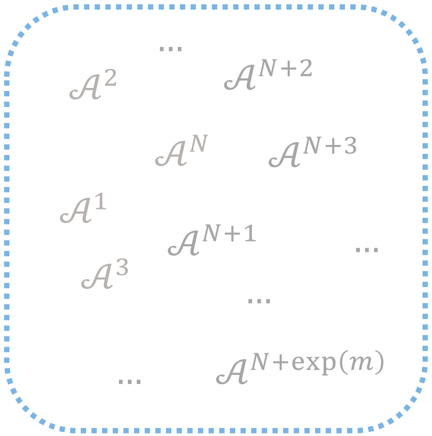

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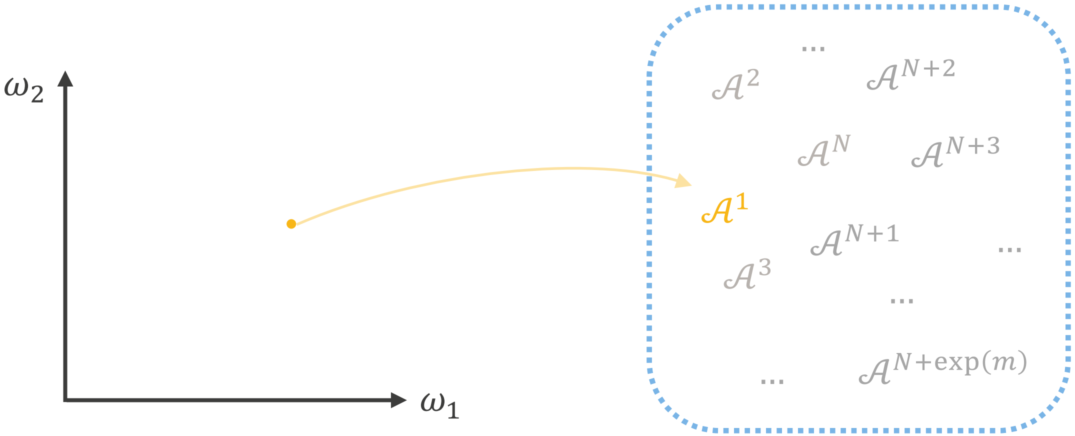

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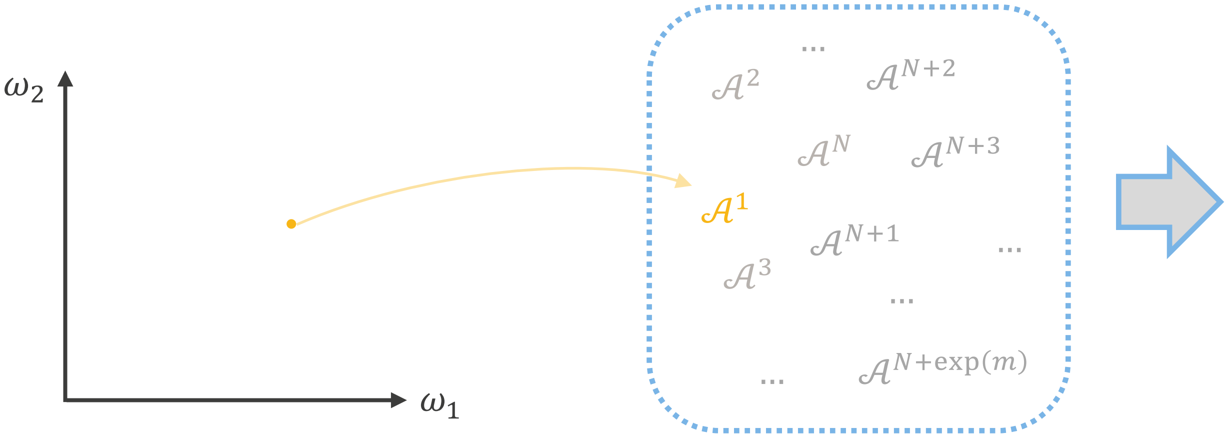

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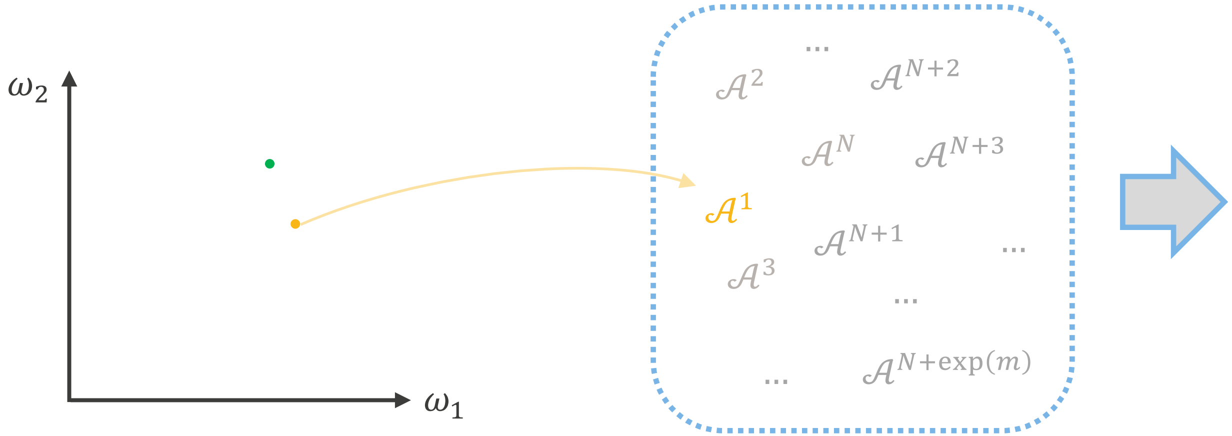
Collection of observed active sets

$$\mathcal{O}_M = \{\mathcal{A}^1\}$$

Learning Collection of Optimal Active Sets



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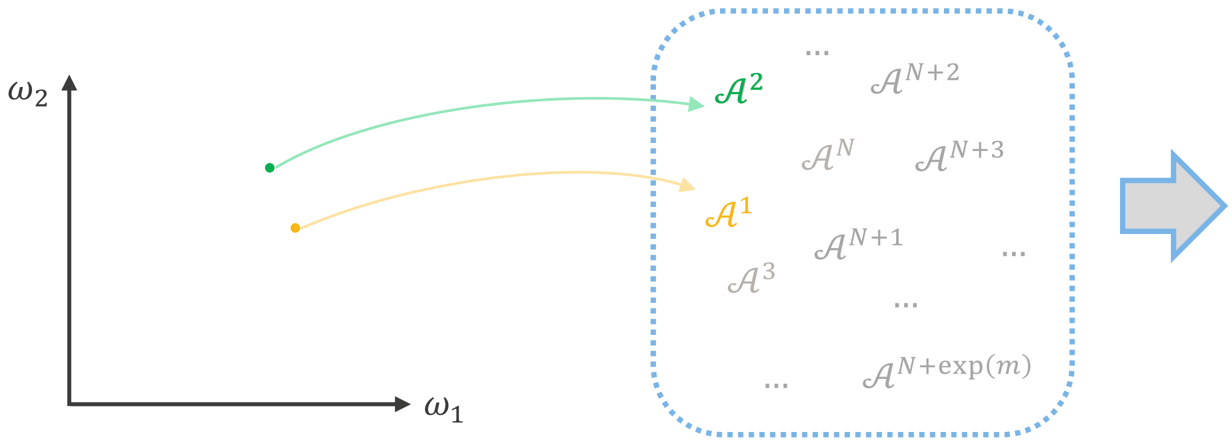

Collection of
observed active sets

$$\mathcal{O}_M = \{\mathcal{A}^1, \mathcal{A}^2\}$$

• samples of input parameters

all possible active sets

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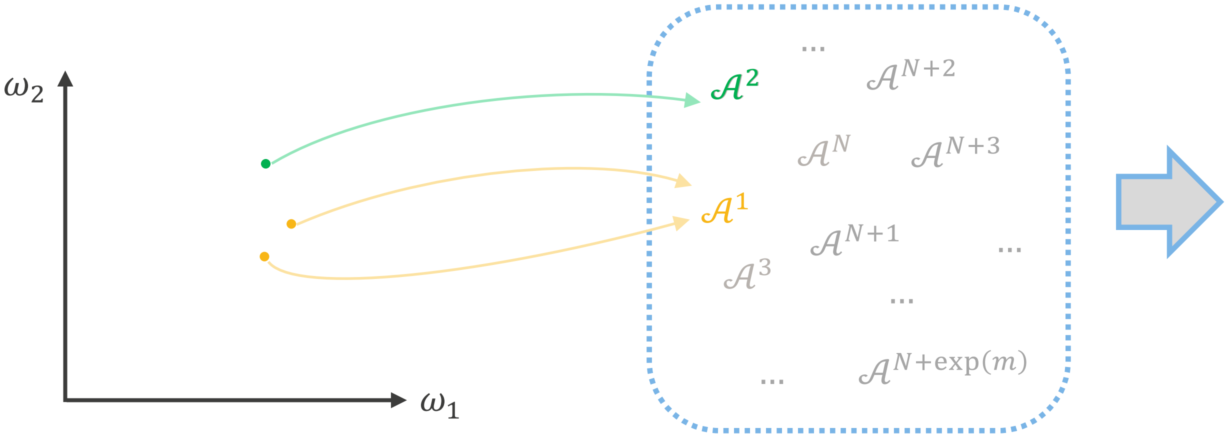
Learning Collection of Optimal Active Sets



- samples of input parameters

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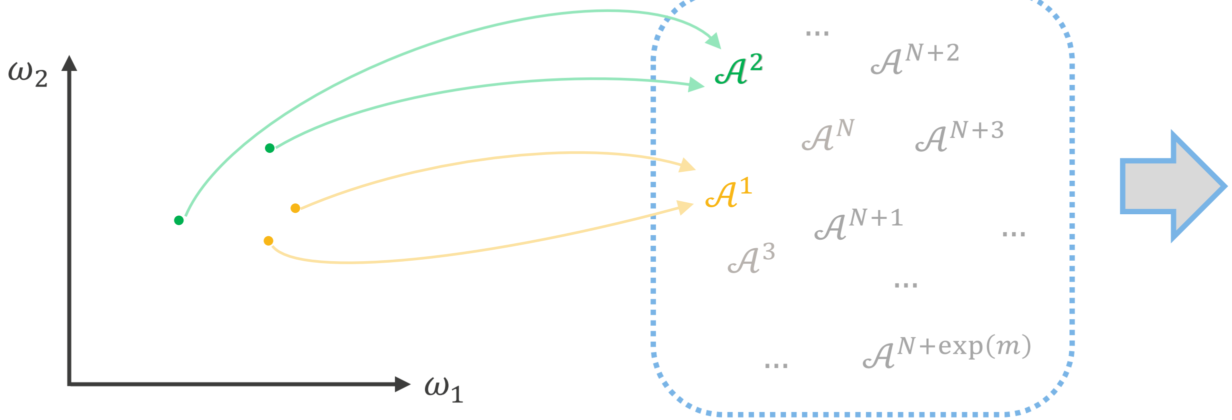
$$\mathcal{O}_M = \{\mathcal{A}^1, \mathcal{A}^2\}$$

Learning Collection of Optimal Active Sets



- samples of input parameters

all possible active sets

color: discovered active sets
grey: undiscovered active sets

Collection of
observed active sets

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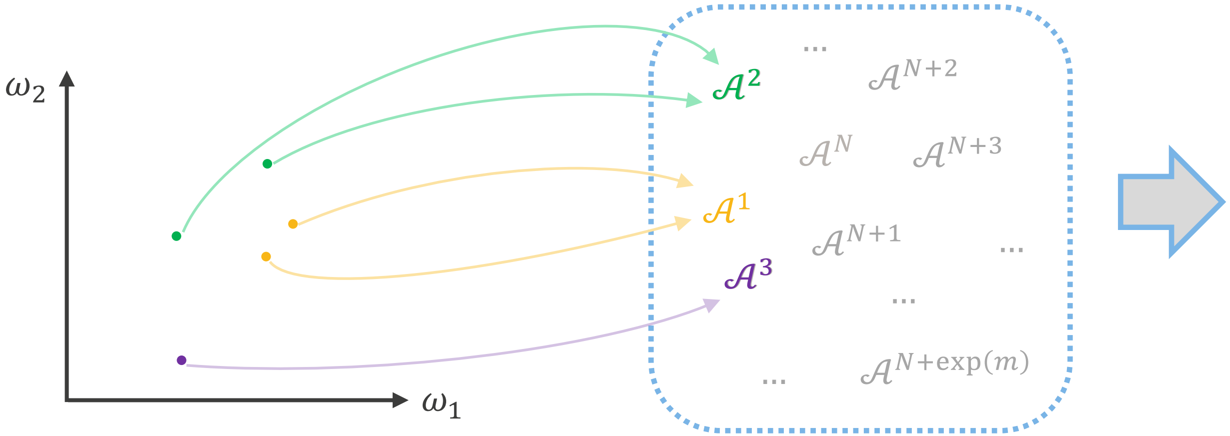
Learning Collection of Optimal Active Sets



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Collection of
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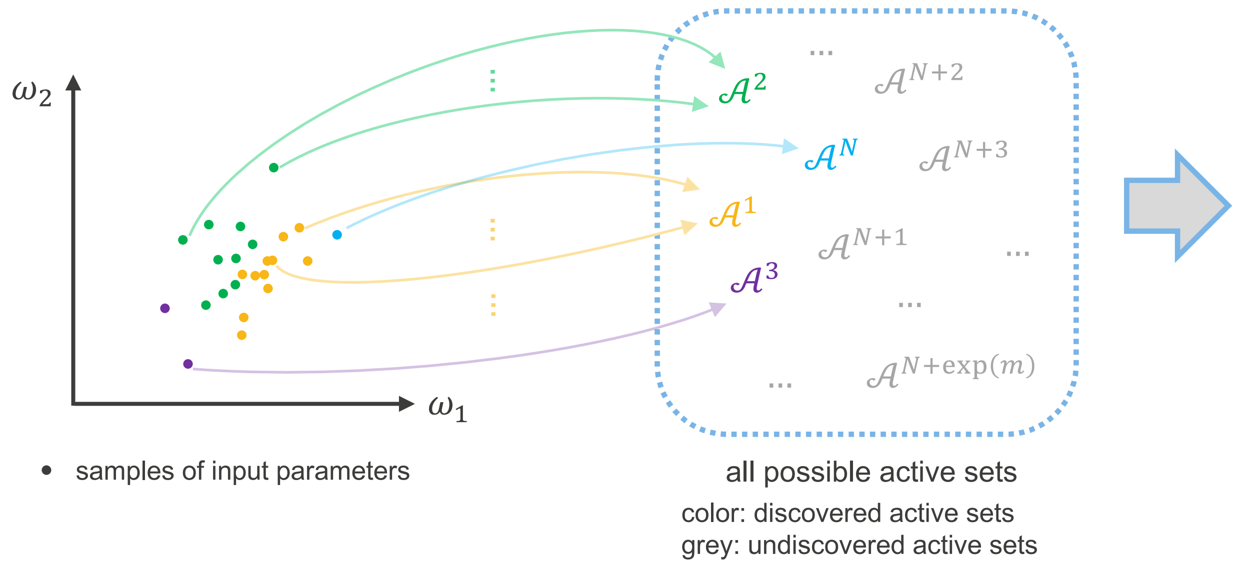
$$\mathcal{O}_M = \{\mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^N\}$$

Learning Collection of Optimal Active Sets



Collection of
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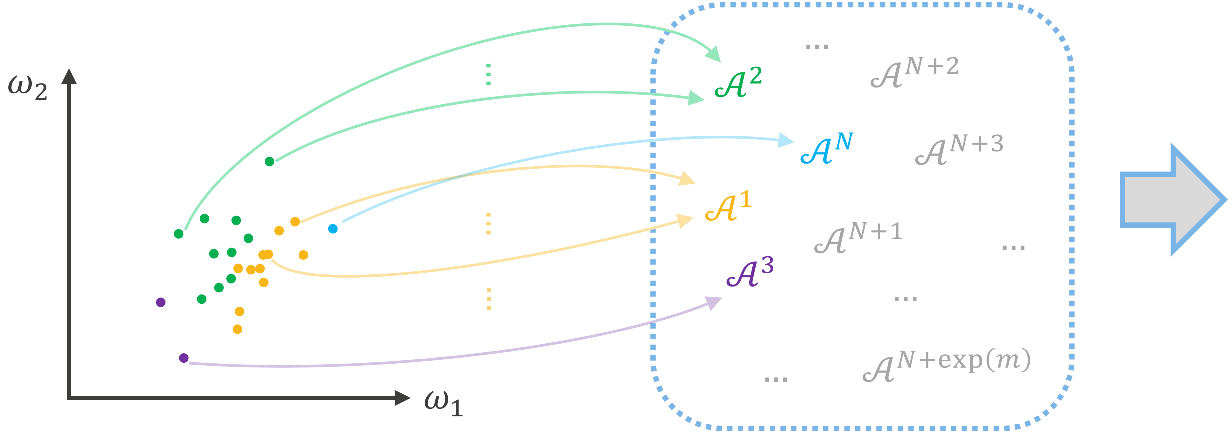
Learning Collection of Optimal Active Sets



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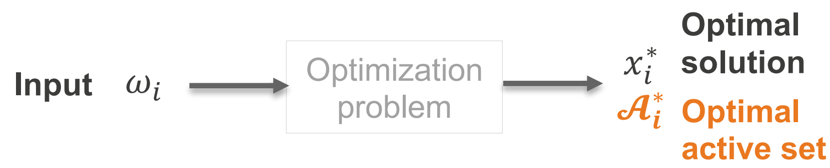
$$\mathcal{O}_M = \{\mathcal{A}^1, \mathcal{A}^2, \dots, \mathcal{A}^N\}$$

When do I stop???

Streaming Algorithm to Learn Collection of Optimal Active Sets

Learning Collection of Optimal Active Sets

Training data:
Renewable energy realizations and corresponding active set $(\boldsymbol{\omega_i}, \boldsymbol{\mathcal{A}_i^*})$



Goal: Find a active sets that together have a high probability of being optimal!

Learning Collection of Optimal Active Sets

1. Observe optimal active sets for M samples

$\mathcal{A}_1^*$ $\mathcal{A}_2^*$ … $\mathcal{A}_M^*$

Collection of **observed** active sets

Learning Collection of Optimal Active Sets

1. Observe optimal active sets for M samples

$\mathcal{A}_1^*$ $\mathcal{A}_2^*$ … $\mathcal{A}_M^*$

Collection of **observed** active sets

2. Check "rate of discovery" for W samples

$\mathcal{A}_1^*$ $\mathcal{A}_2^*$ … $\mathcal{A}_W^*$

How frequently do we observe sets
we have **not seen** before?

Rate of discovery: $\boldsymbol{R_{M,W}} = \frac{N_{unobserved}}{N}$

where $W = \frac{2\gamma}{\epsilon^2}\max\{\log(M), \log(\underline{M})\}$

$$\underline{M} = 1 + \left(\frac{\gamma}{\delta(\gamma-1)}\right)^{\frac{1}{\gamma-1}}$$

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$$\underline{M} = 1 + \left(\frac{\gamma}{\delta(\gamma-1)}\right)^{\frac{1}{\gamma-1}}$$

- If the rate of discovery is below the threshold $R_{M,W} \leq \alpha - \epsilon$, stop.

performance guarantee

Learning Collection of Optimal Active Sets

1. Observe optimal active sets for M samples

$\mathcal{A}_1^*$ $\mathcal{A}_2^*$ … $\mathcal{A}_M^*$ $\mathcal{A}_{M+1}^*$

Collection of **observed** active sets

2. Check “rate of discovery” for W samples

$\mathcal{A}_1^*$ $\mathcal{A}_1^*$ … $\mathcal{A}_W^*$ $\mathcal{A}_{W+1}^*$

How frequently do we observe sets we have **not seen** before?

- If the rate of discovery is below the threshold $R_{M,W} \leq \alpha - \epsilon$, stop.
- If the rate of discovery is too high, add more samples.

Guarantees performance at termination

[Misra, Roald, Ng, 2018]

Learning Collection of Optimal Active Sets

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[Misra, Roald, Ng, 2018]

Guaranteed to converge!

Learning Collection of Optimal Active Sets

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Collection of **observed** active sets

2. Check "rate of discovery" for W samples

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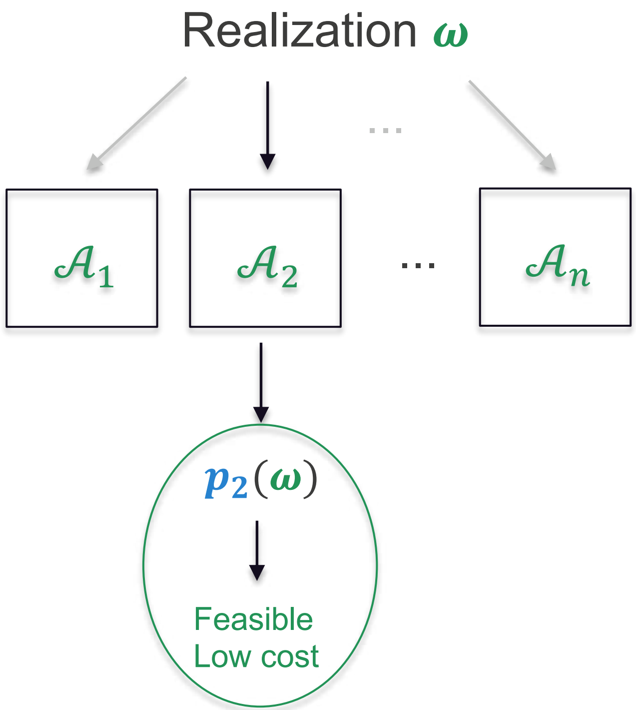
Guaranteed to converge fast for low number of optimal active sets! [Misra, Roald, Ng, 2018]

Practicability of the approach



Simultaneously establishes

- Collection of optimal active sets
- Practicability of the approach

No assumptions on distribution

No assumptions on problem structure

Guaranteed to converge fast for low number of optimal active sets!

Results for the (linear)

DC Optimal Power Flow Problem

Streaming Algorithm Results – PGLib-OPF v 17.08

Probabilistic guarantee: $\mathbb{P}_\omega\big(\pi(\mathcal{U}_M)\big) < \alpha = 0.05,$

Max. difference: $\epsilon = 0.04$

Confidence level: $\delta = 0.01$

Hyperparameter: $\gamma = 2$

Termination: $R_{M,W} \leq 0.01$

$W = \frac{2\gamma}{\epsilon^2}\max\{\log M, \log \underline{M}\}$ with $\underline{M} = 1 + \left(\frac{\gamma}{\delta(\gamma-1)}\right)^{\frac{1}{\gamma-1}}$

Initial W: $W = 13'259$

(constant until $M = 201$)

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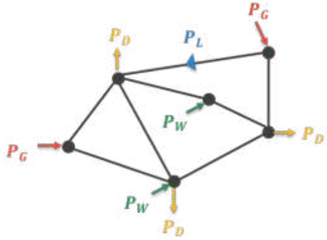
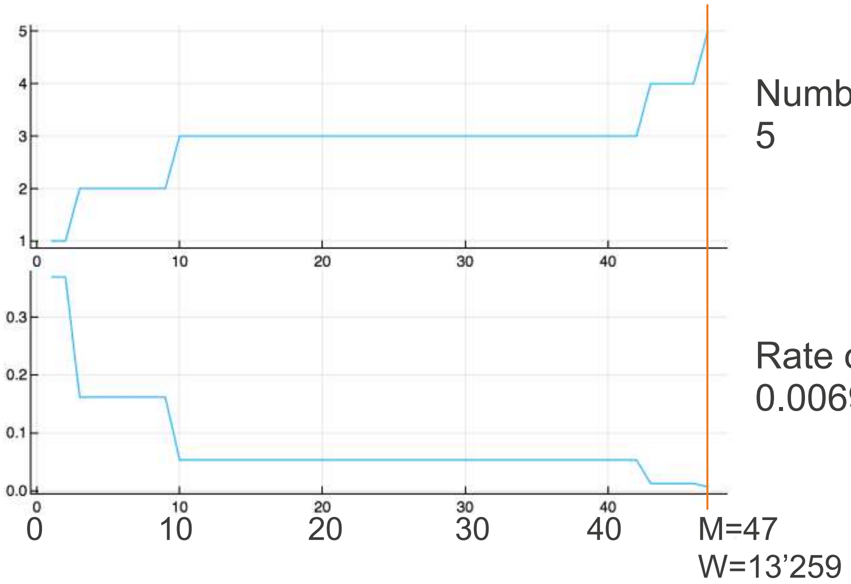


Uncertain loads:

Normal distribution
$\omega \sim \mathcal{N}(0, \sigma = 0.03d)$

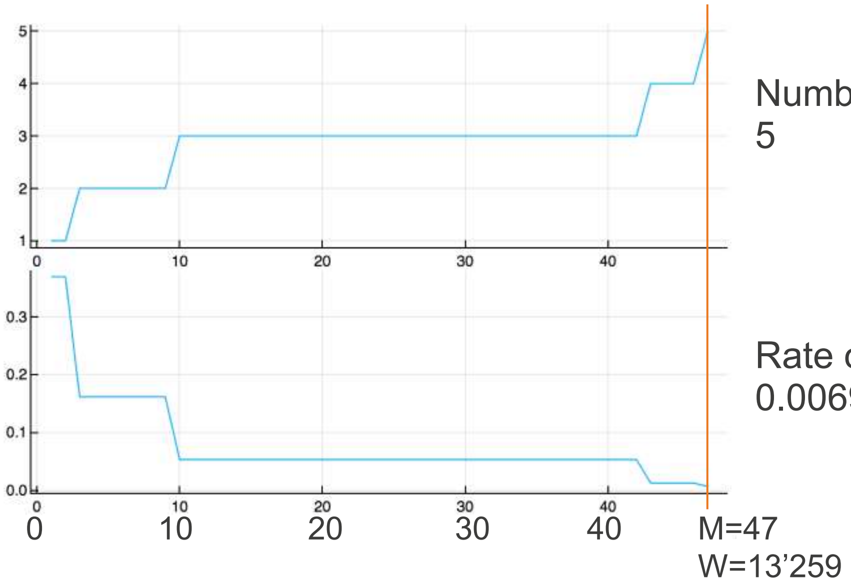
Uniform distribution
with support
$\omega \in [-0.09d, 0.09d]$

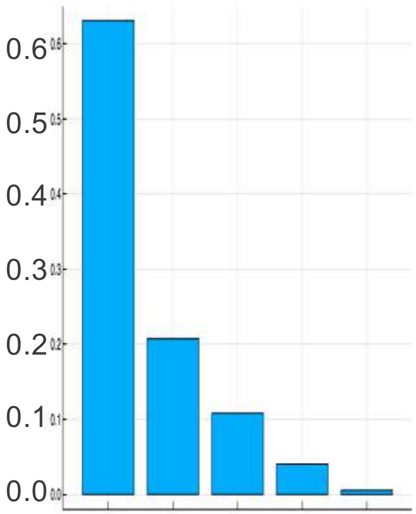
Example – RTE 1951 bus test case



When there are few relevant active sets, the algorithm terminates fast!

Example – PSERC 200 bus test case





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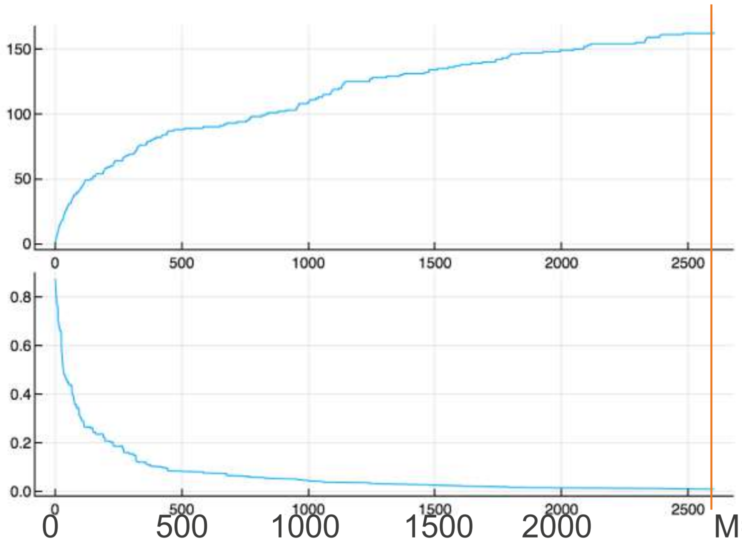
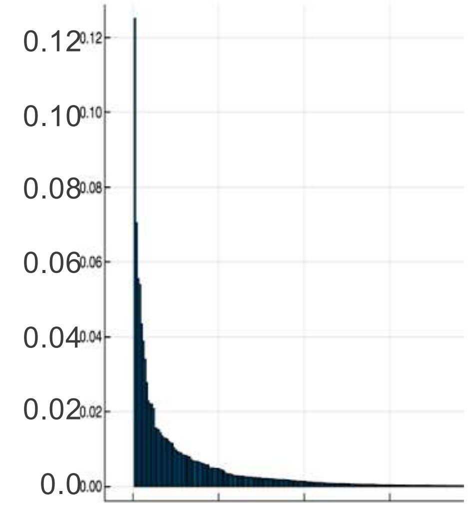


Number of active sets
163

Rate of discovery
0.0099

M=2602
W=19’661



Probability of observed sets

When there are many relevant active sets, the algorithm terminates slowly!

Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$ ➔ Termination: $R_{M,W} \leq 0.01$

	Normal distribution					Uniform distribution				
	K_M	M	W_M	$R_{M,W}$	$\mathbb{P}(p^*)$	K_M	M	W_M	$R_{M,W}$	$\mathbb{P}(p^*)$
Low-Complexity										
case3_lmbd	1	1	13'259	0.0	1.0	1	1	13'259	0.0	0.0
case5_pjm	1	1	13'259	0.0	1.0	1	1	13'259	0.0	0.0
case14_ieee	1	1	13'259	0.0	1.0	1	1	13'259	0.0	0.0
case30_ieee	1	1	13'259	0.0	1.0	1	1	13'259	0.0	0.0
case39_epri	2	2	13'259	0.0	1.0	2	2	13'259	0.0008	0.9998
case118_ieee	2	33	13'259	0.0	1.0	2	4	13'259	0.0019	0.9984
case57_ieee	2	2	13'259	0.0003	0.9997	3	46	13'259	0.0	0.0
case1888_rte	3	6	13'259	0.0	0.0	3	10	13'259	0.0	0.0
case1951_rte	5	47	13'259	0.0069	0.9943	11	63	13'259	0.0084	0.9901
case162_ieee_dtc	7	91	13'259	0.0054	0.9925	17	192	13'259	0.0085	0.9926
case24_ieee_rts	10	1456	18'209	0.0	0.0	11	64	13'259	0.0047	0.9941
High-Complexity										
case73_ieee_rts	19	1258	17'844	0.0087	0.9931	130	22'000	24'977	0.0136	-
case300_ieee	24	1257	17'842	0.0073	0.9919	293	9095	22'789	0.0099	0.9897
case200_pserc	174	4649	21'112	0.0099	0.9909	236	6741	22'040	0.0099	0.9901
case240_pserc	2993	22'000	24'997	0.0795	-	2993	22'000	24'997	0.0795	-

System sizes ranging from 3 to 1951 nodes

Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$ ⇨ Termination: $R_{M,W} \leq 0.01$

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with support
$\omega \in [-0.09d, 0.09d]$

Streaming Algorithm Results – PGLib-OPF v 17.08

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Streaming Algorithm Results – PGLib-OPF v 17.08

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Streaming Algorithm Results – PGLib-OPF v 17.08

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Few active sets!

Terminates fast.

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Few active sets!

Terminates fast.

High probability of optimal solutions!

Streaming Algorithm Results – PGLib-OPF v 17.08

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case30_ieee	1	1	13'259	0.0	1.0	1	1	13'259	0.0	1.0
case39_epri	2	2	13'259	0.0	1.0	2	2	13'259	0.0008	0.9998
case118_ieee	2	33	13'259	0.0	1.0	2	4	13'259	0.0019	0.9984
case57_ieee	2	2	13'259	0.0003	0.9997	3	46	13'259	0.0	1.0
case1888_rte	3	6	13'259	0.0	1.0	3	10	13'259	0.0	1.0
case1951_rte	5	47	13'259	0.0069	0.9943	11	63	13'259	0.0084	0.9901
case162_ieee_dtc	7	91	13'259	0.0054	0.9925	17	192	13'259	0.0085	0.9926
case24_ieee_rts	10	1456	18'209	1.0	1.0	11	64	13'259	0.0047	0.9941
High-Complexity										
case73_ieee_rts	19	1258	17'844	0.0087	0.9931	130	22'000	24'977	0.0136	-
case300_ieee	24	1257	17'842	0.0073	0.9919	293	9095	22'789	0.0099	0.9897
case200_pserc	174	4649	21'112	0.0099	0.9909	236	6741	22'040	0.0099	0.9901
case240_pserc	2993	22'000	24'997	0.0795	-	2993	22'000	24'997	0.0795	-

Few active sets!

Terminates fast.

High probability of optimal solutions!

Not always:

Large number of active sets

Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$ ⇨ Termination: $R_{M,W} \leq 0.01$

	Normal distribution					Uniform distribution				
	K_M	M	W_M	$R_{M,W}$	$\mathbb{P}(p^*)$	K_M	M	W_M	$R_{M,W}$	$\mathbb{P}(p^*)$
Low-Complexity										
case3_lmbd	1	1	13'259	0.0	1.0	1	1	13'259	0.0	1.0
case5_pjm	1	1	13'259	0.0	1.0	1	1	13'259	0.0	1.0
case14_ieee	1	1	13'259	0.0	1.0	1	1	13'259	0.0	1.0
case30_ieee	1	1	13'259	0.0	1.0	1	1	13'259	0.0	1.0
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Few active sets!

Terminates fast.

High probability of optimal solutions!

Not always:

Large number of active sets

Slow to terminate.

Streaming Algorithm Results – PGLib-OPF v 17.08

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case240_pserc	2993	22'000	24'997	0.0795	-	2993	22'000	24'997	0.0795	-

Few active sets!

Terminates fast.

High probability of optimal solutions!

Not always:

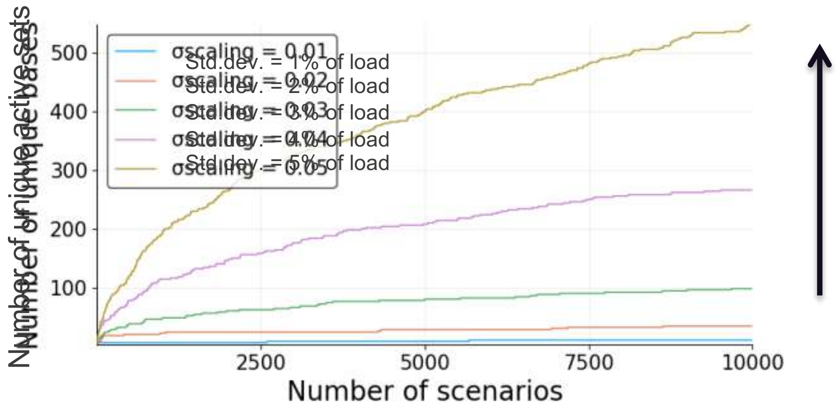
Large number of active sets

Slow to terminate.

Lower probability of optimal solutions!

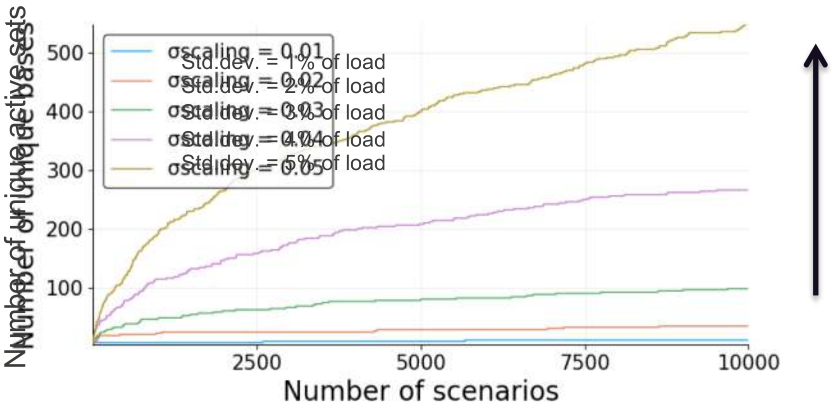
Practical Implications for Power Systems Operation

IEEE 300 bus system with normally distributed load



Increasing parameter uncertainty =
Increasing number of optimal active sets

IEEE 300 bus system with normally distributed load

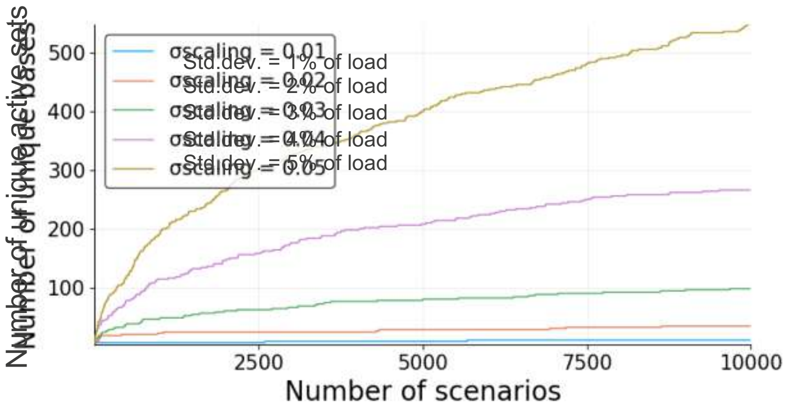


Increasing parameter uncertainty =
Increasing number of optimal active sets

«Power systems operation becomes more unpreditable and complex with increasing uncertainty»

General perception among system operators

IEEE 300 bus system with normally distributed load



Increasing parameter uncertainty =
Increasing number of optimal active sets

«Power systems operation becomes more unpreditable and complex with increasing uncertainty»

General perception among system operators

What does this imply for system risk? Price stability?

Summary

 # 1 – Leverage pre-existing knowledge (mathematical model) improves learning outcomes

 # 2 – Using active sets as an intermediate step is useful

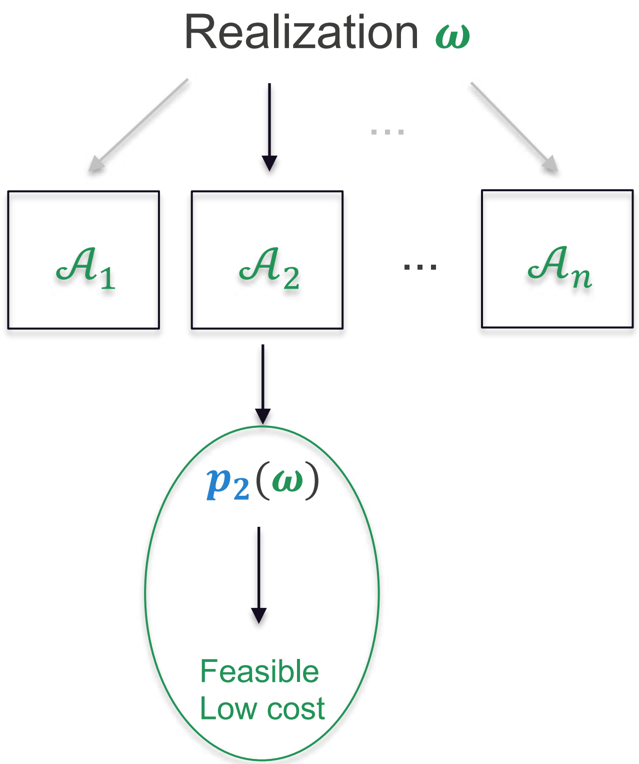
- encodes all information about optimal solution
- finite (and typically low?) number of active sets

 # 3 – Streaming algorithm establishes practicability of the task

- Probabilistic performance guarantees
- Guaranteed to terminate
- Guaranteed to terminate fast for nice problems

- **Quite general strategy**
  - Streaming algorithm can work for very general problems: Non-convex AC OPF, mixed integer problems …
  - Disclaimer: Application must be such that the number of active sets is small.
  - Alternative strategy: Learn possible *active constraints* instead of active sets

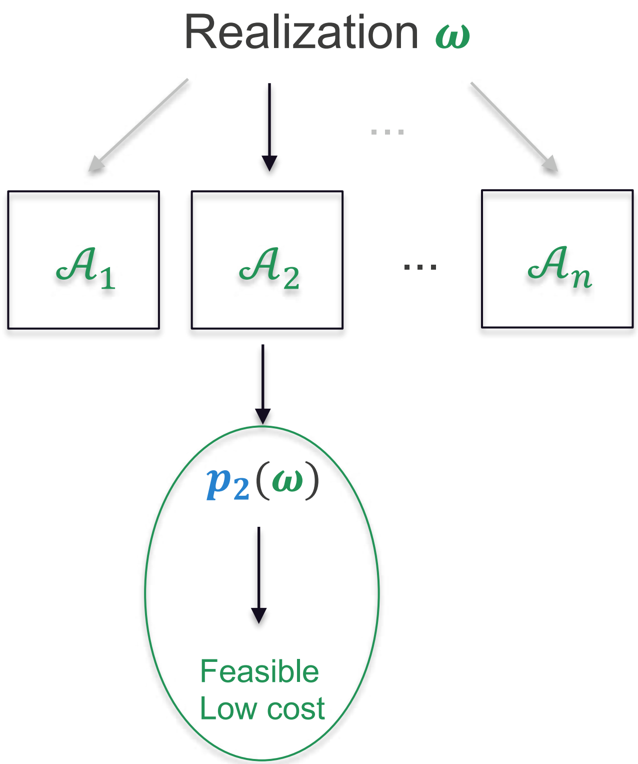
Outlook



Classification!

[Deka and Misra, 2019]

Outlook



Classification!

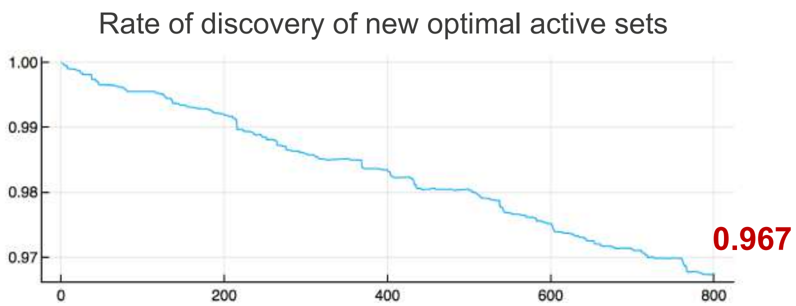
Efficient solution?
Active set solver, local approximation, …

Preliminary results for the (non-linear, non-convex) AC Optimal Power Flow Problem

AC Optimal Power Flow

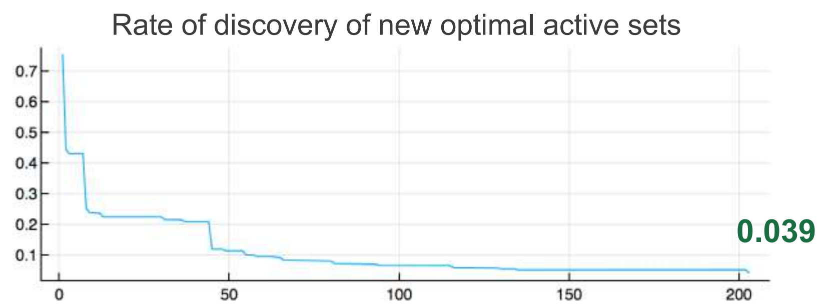
Max. undiscovered: $\alpha = 0.1$, Max. difference: $\epsilon = 0.05$ ➡ Termination: $R_{M,W} \leq 0.05$

RTE 1951 bus test case



(did not terminate)

PSERC 200 bus test case

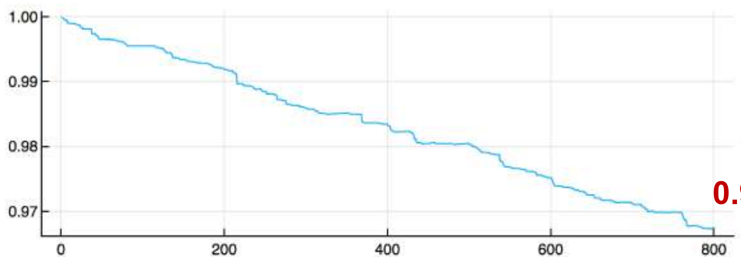


AC Optimal Power Flow

Max. undiscovered: $\alpha = 0.1$, Max. difference: $\epsilon = 0.05$ ⇨ Termination: $R_{M,W} \leq 0.05$

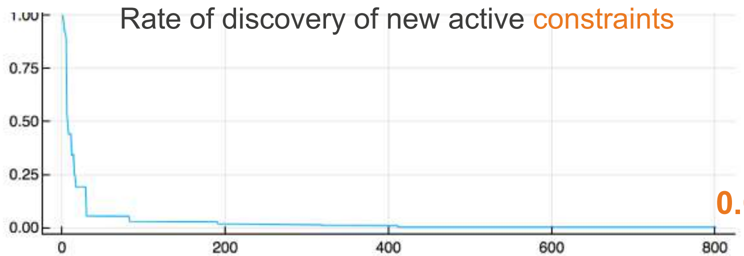
RTE 1951 bus test case

Rate of discovery of new optimal active sets



0.967

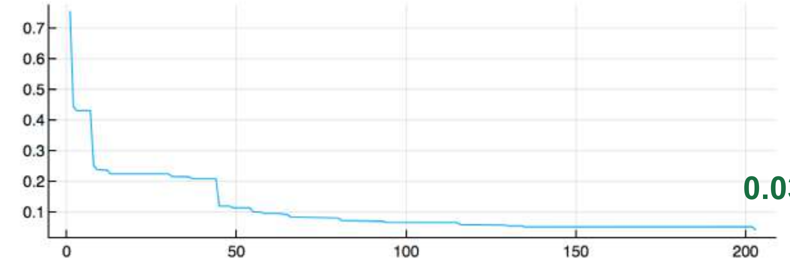
Rate of discovery of new active constraints



0.0035

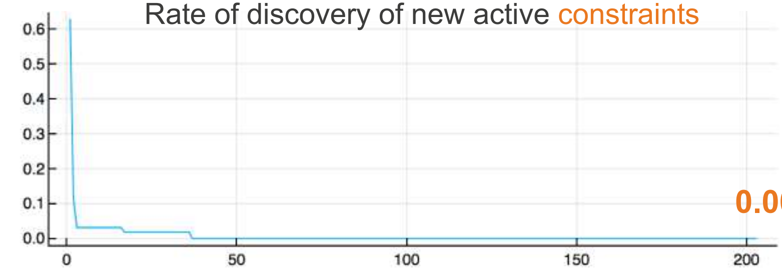
PSERC 200 bus test case

Rate of discovery of new optimal active sets



0.039

Rate of discovery of new active constraints



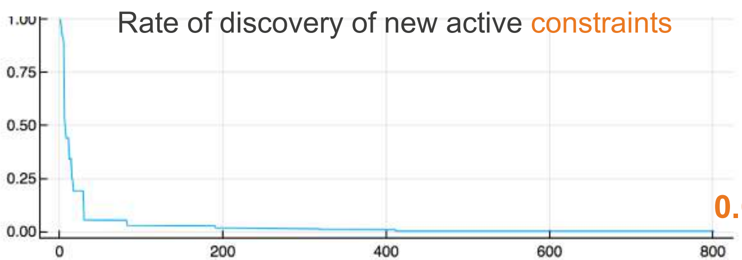
0.0001

AC Optimal Power Flow

Max. undiscovered: $\alpha = 0.1$, Max. difference: $\epsilon = 0.05$ → Termination: $R_{M,W} \leq 0.05$

RTE 1951 bus test case

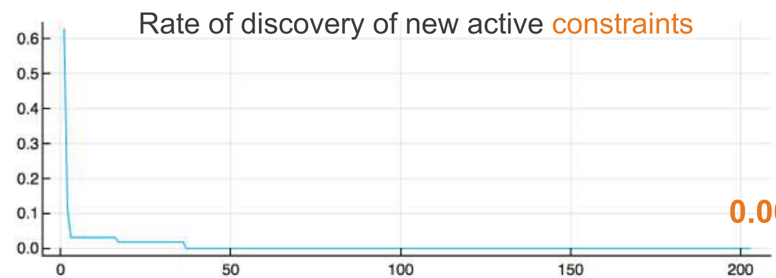
Only 164 of 5192 transmission line constraints ever active



0.0035

PSERC 200 bus test case

Only 28 of 490 transmission line constraints ever active



0.0001

1. Learning solutions to (power system) optimization problems through optimal active sets

2. **Identifying potentially active constraints**

Identifying potentially active constraints

Daniel K. Molzahn

Georgia Tech

Dan Molzahn and Line Roald, «Grid-Aware versus Grid-Agnostic Distribution System Control: A Method for Certifying Engineering Constraint Satisfaction», Hawaii International Conference on System Sciences (HICSS), 2019

Line Roald and Dan Molzahn, «Implied Constraint Satisfaction in Power System Optimization: The Impacts of Load Variations», available online: https://arxiv.org/abs/1904.01757

Optimal Power Flow

$$\min_{p_G(\omega)} \quad \sum_{i\in\mathcal{G}}\left(c_{2,i}p_{G,i}(\omega)^2 + c_{1,i}\,p_{G,i}(\omega)\right)$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

$$p_{G,g}^{min} \leq p_{G,g}(\omega) \leq p_{G,g}^{max}, \ \ g \in \mathcal{G}$$
$$q_{G,g}^{min} \leq q_{G,g}(\omega) \leq q_{G,g}^{max}, \ \ g \in \mathcal{G}$$

$$v_i^{min} \leq v_i(\omega) \leq v_i^{max}, \quad i \in \mathcal{B}$$

$$i_{L,j}(\omega) \leq i_{L,j}^{max}, \quad j \in \mathcal{L}$$

Before, we learned constraints that are *likely* to be active

Feasible set in the direction of the **cost function**

Now we want to understand which constraints can *possibly* be active!

The full **feasible set**

Optimization-based constraint screening

Main idea:
Minimize/maximize the value of the constraints!

$\min \quad v_i \;\; / \max \quad v_i$

Minimize/maximize voltage, currents …

s.t.

$f(\theta, v, p, q) = 0,$

Non-Linear AC Power Flow

$p_{G,g}^{min} \leq p_{G,g} \leq p_{G,g}^{max}, \;\; g \in \mathcal{G}$
$q_{G,g}^{min} \leq q_{G,g} \leq q_{G,g}^{max}, \;\; g \in \mathcal{G}$

Generation constraints

$v_i^{min} \leq v_i \leq v_i^{max}, \qquad i \in \mathcal{B}$

Voltage constraints

$s_{L,j}\,(\theta, v, p, q) \leq s_{L,j}^{max}, \;\; j \in \mathcal{L}$

Transmission constraints

If $\max v_i \leq v_i^{max}$
and $\min \; v_i \geq v_i^{min}$

Voltage will never go out of bounds!

Optimization-based constraint screening

Main idea:
Minimize/maximize the value of the constraints!

$\min \quad v_i \;\; / \max \quad v_i$	**Minimize/maximize voltage, currents …**
s.t.	
$f(\theta, v, p, q) = 0,$	**Non-Linear AC Power Flow**
$p_{G,g}^{min} \leq p_{G,g} \leq p_{G,g}^{max}, \;\; g \in \mathcal{G}$ $q_{G,g}^{min} \leq q_{G,g} \leq q_{G,g}^{max}, \;\; g \in \mathcal{G}$	**Generation constraints**
$v_i^{min} \leq v_i \leq v_i^{max}, \quad i \in \mathcal{B}$	**Voltage constraints**
$s_{L,j}\,(\theta, v, p, q) \leq s_{L,j}^{max}, \;\; j \in \mathcal{L}$	**Transmission constraints**

- Connections to optimization-based bound tightening
  [C. Coffrin, Hijazi, and Van Hentenryck, 2015]
- Results for DC OPF
  [Ardakani and F. Bouffard, 2013, 2015]
  [Madani, Lavaei, and Baldick, 2017]
- Our interest:
  - Large ranges of load
  - AC OPF (distribution grids)

Distribution grids – AC Optimal Power Flow

- Consider ranges of **load variations (not controllable by the system operator)**
- Voltage constraints only on buses we **monitor/control**

$$\min \quad v_i \;\; / \max \quad v_i$$

s.t.

$$f(\theta, v, p, q) = 0,$$

$$p_{D,i}^{min} \leq p_{D,i} \leq p_{D,i}^{max}, \; i \in N$$
$$q_{D,i}^{min} \leq q_{D,i} \leq q_{D,i}^{max}, \; i \in N$$

$$v_i^{min} \leq v_i \leq v_i^{max}, \quad i \in C$$

Minimize/maximize voltage

~~**Non-Linear AC Power Flow**~~

Load variations

Voltage constraints on nodes with measurements/control

Valid bounds:
Use convex relaxation.

QC relaxation with bound tightening

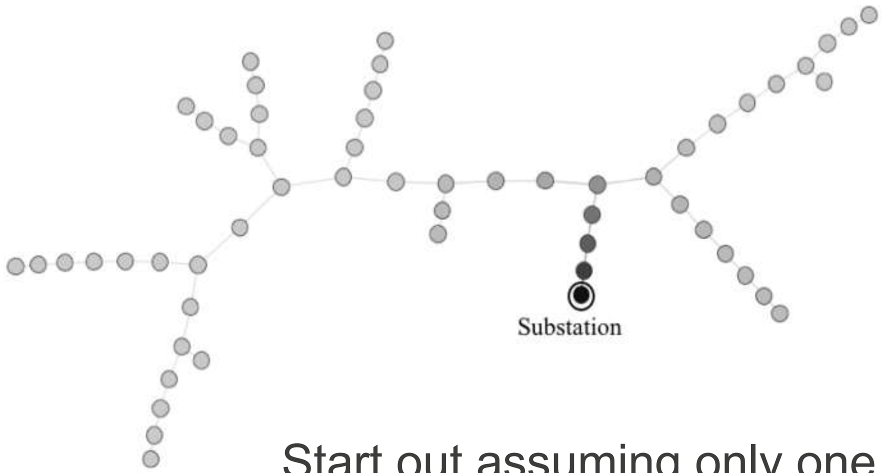
[Coffrin, Hijazi and Van Hentenryck, 2016 & 2017]

Challenging:
- non-standard objective (relaxation is weak)
- low-voltage solutions
- …

Can we certify safe operations?

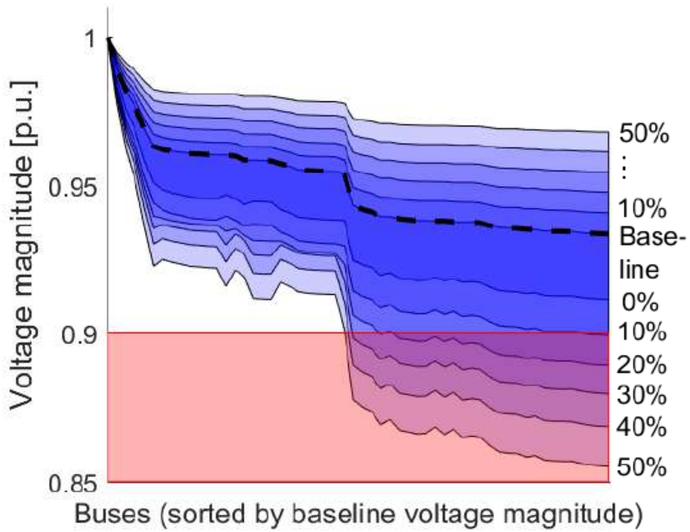
IEEE 123 bus system –
single-phase equivalent

[Bolognani and Zampieri, 2016]

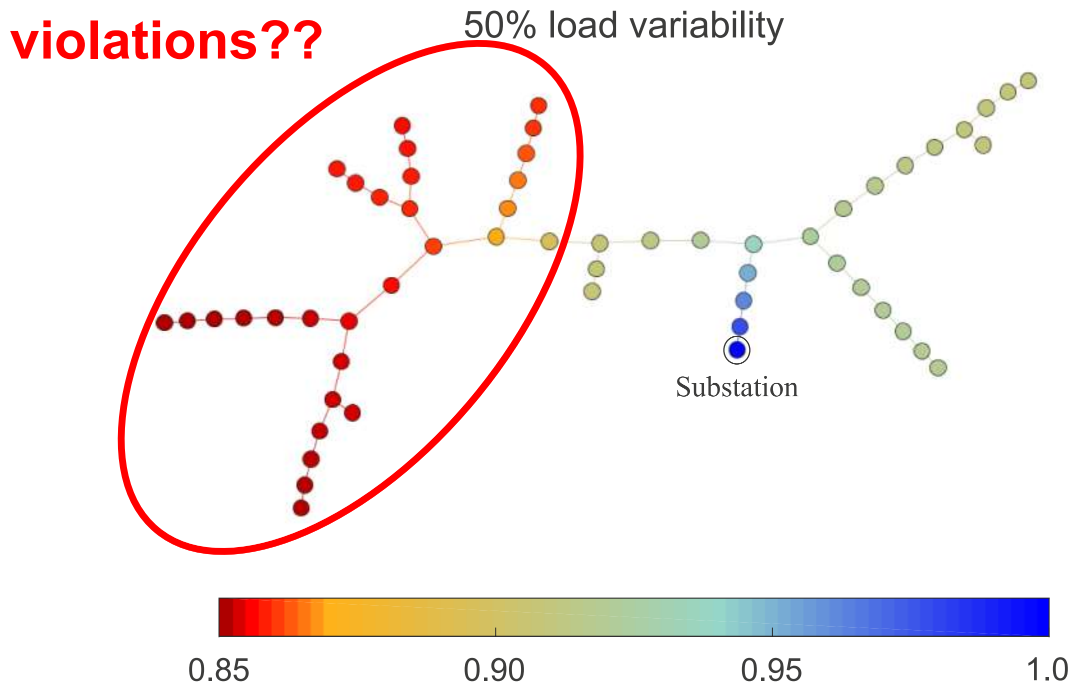


Start out assuming only one node with controlled voltage

Can we certify safe operations?

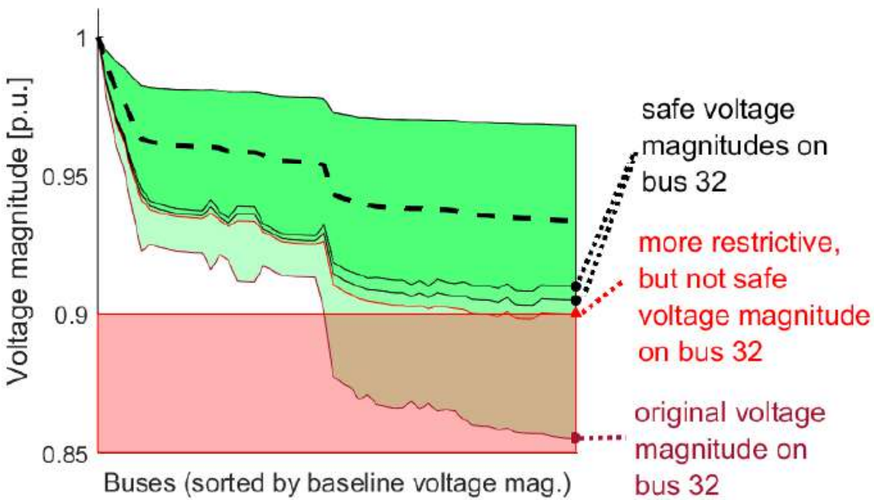


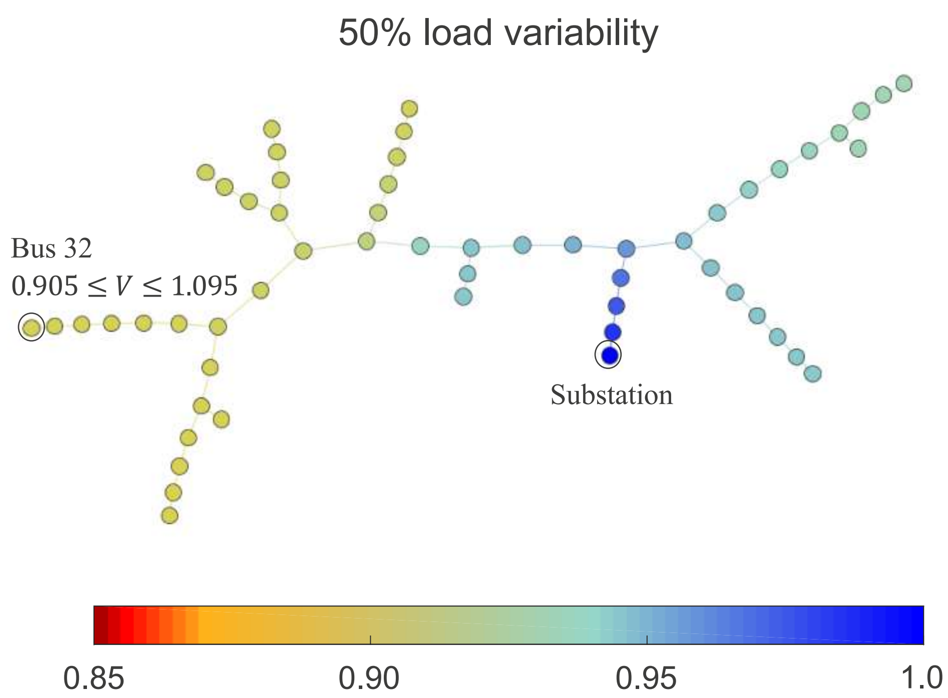
Increasing load variability, increasing voltage range!



Add more controllable nodes, and tighten the voltage limits!

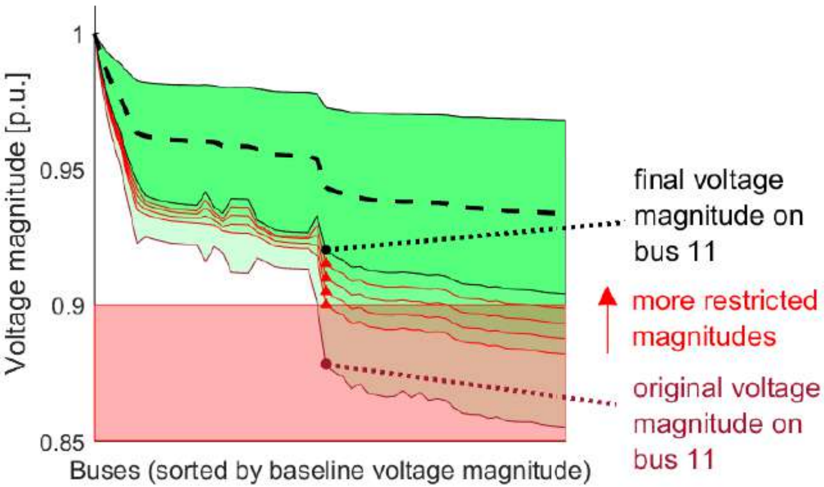
Can we certify safe operations?

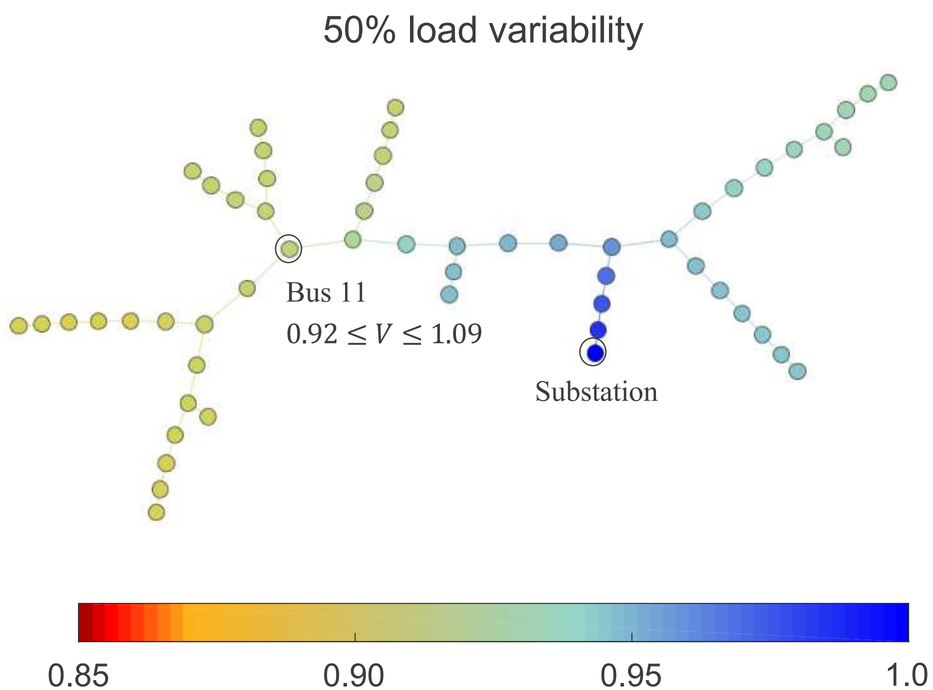




Added controllability and tighter voltage range on Bus 32

Can we certify safe operations?





Added controllability and tighter voltage range on Bus 11

Redundant constraints in DC optimal power flow

How many constraints can ever be active in DC optimal power flow?

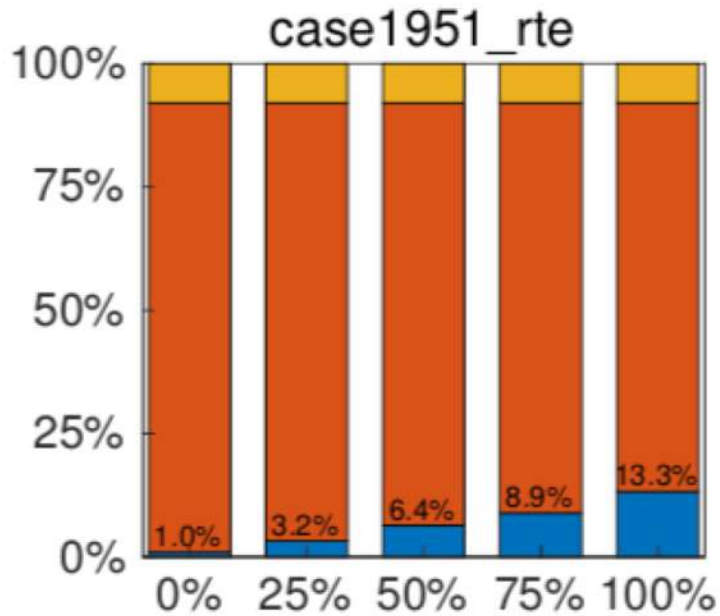
$\min_{P_G} \quad C_G^T P_G$	Minimize/maximize constraints	
s.t. $\quad \sum_{i=1}^{N_B}\left(P_{G(i)} - \boldsymbol{P_{D(i)}}\right) = 0$	Power balance	Non-redundant
$\mathbf{0} \leq P_G \leq P_G^{max}$,	Generation constraints	Non-redundant
$-P_L^{max} \leq M(P_G - \boldsymbol{P_D}) \leq P_L^{max}$,	Transmission constraints	**Often redundant**

Allow power demand $\boldsymbol{P_{D(i)}}$ to vary $\pm X$ % where $0 \leq X \leq 100$

Relax generator lower bounds to $\mathbf{0}$

Results on PGLib-OPF test cases

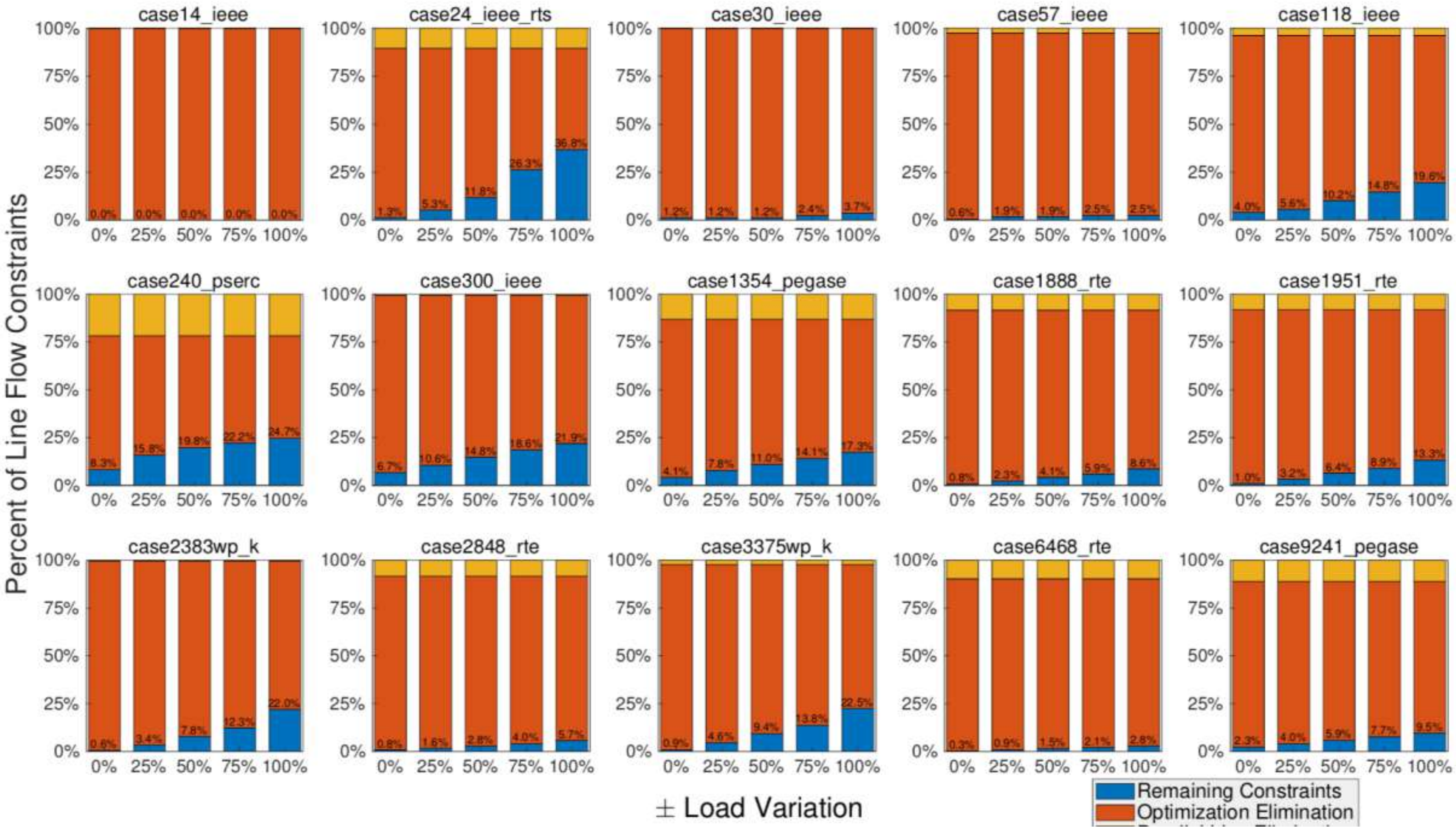
Percentage of line flow constraints



$\pm$ % Load variation

Remaining non-redundant constraints

Results on standard test cases



Optimization-based constraint screening

- **Main idea:**
  - Solve optimization problems that minimize/maximize the value of the constraints (If the problems are hard to solve, use relaxations to obtain valid lower/upper bounds!)
  - Identify constraints that cannot be violated -> redundant constraints
  - Identify constraints that can be violated -> potentially important constraints

- **Works really well for power flow optimization!**

- **We can use this to**
  (1) identify constraints that need to be monitored/controlled
  (2) reduce the number of considered constraints
  (3) …

THANK YOU!

Line Roald, roald@wisc.edu

Summary of streaming algorithm results

1. **Guaranteed to terminate, no need to decide on the number of M samples apriori**
   Definition of the window size W and termination criterion

Theorem 1 and 2 [Misra, Roald, Ng, 2018]:

If the window size $W(M)$ is defined as

$$W = \frac{2\gamma}{\epsilon^2} \max\{\log M, \log \underline{M}\} \text{ with } \underline{M} = 1 + \left(\frac{\gamma}{\delta(\gamma-1)}\right)^{\frac{1}{\gamma-1}}$$

Then $\mathbb{P}\left(\pi(U_M) - R_{M,W} \leq \epsilon \;\; \forall M > 1\right) \leq 1 - \delta$

ϵ difference between true and empirical probability of unobserved active sets

δ confidence level

γ hyperparameter > 1

2. **Guaranteed to terminate fast for *benign* systems**

Theorem 3 [Misra, Roald and Ng] If a (small) number of relevant active sets K_0 that contains more than $1 - \alpha_0$ probability mass, then with probability at least $1 - \delta - \delta_0$ the algorithm terminates in less iterations than

$$M = \frac{1}{\alpha - \alpha_0}\left(K_0 \log 2 + \log \frac{1}{\delta_0}\right)$$