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Data-Driven Recovery of Frequency Response from Ambient Synchrophasor Data

Hao Zhu

Assistant Professor
Department of ECE
The University of Texas at Austin

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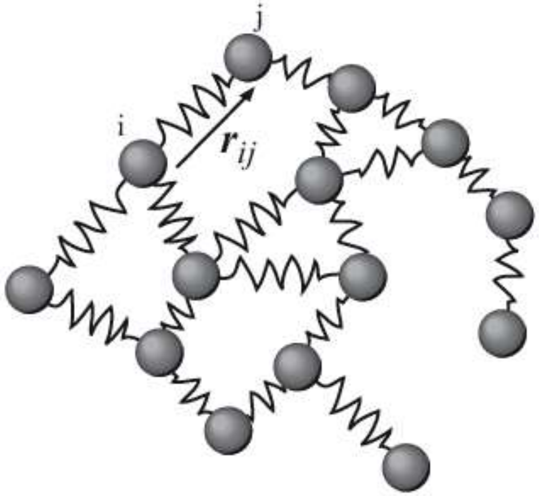
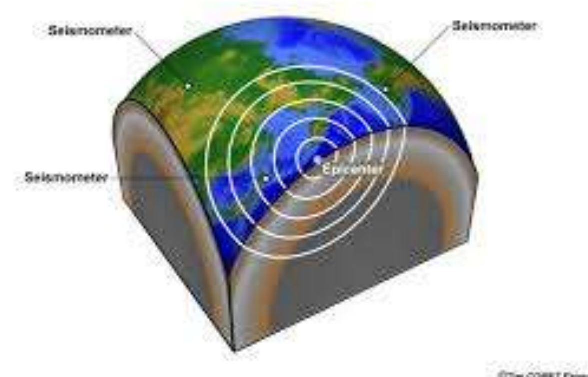
Oscillatory Networks



$\ddot{x}_i = \sum k_{ij} x_j$

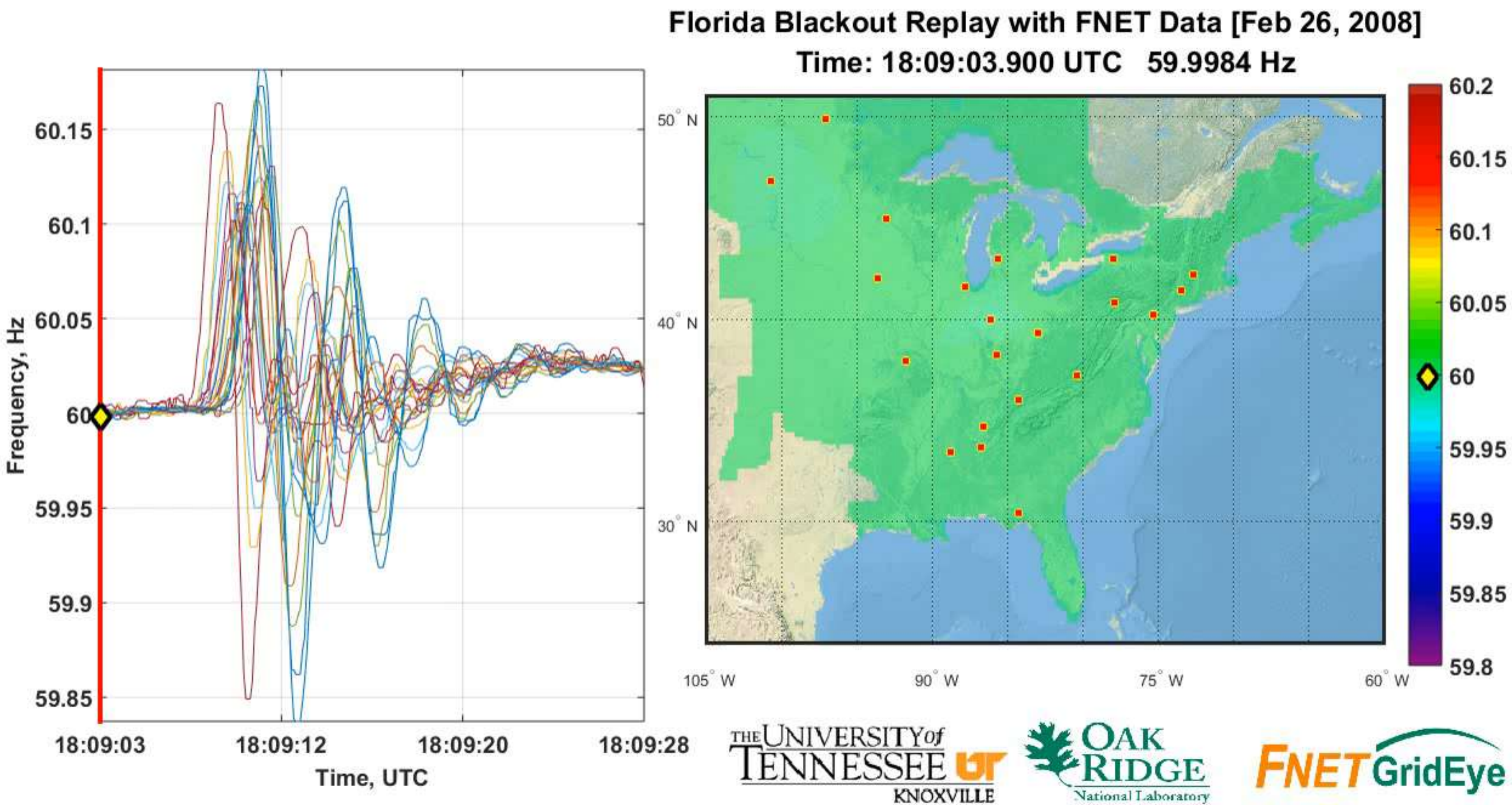
$\mathbf{M}\ddot{\boldsymbol{\delta}} + \mathbf{D}\dot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} = \mathbf{u}$



$\frac{\partial^2 u}{\partial t^2} + r\frac{\partial u}{\partial t} = c^2 \nabla^2 u$

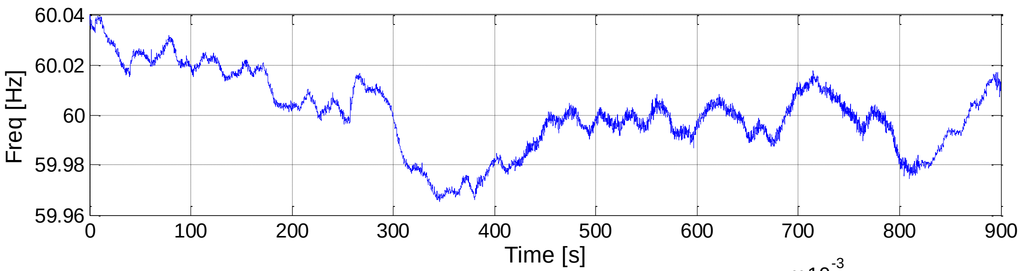
- Many natural (even societal) networks have oscillatory dynamics
- Sensors ubiquitous in actual networked systems
  - Collecting huge volume of data during normal conditions (small perturb.)
  - Phasor measurement unit (PMU) in power grids
  - Seismometers installed around the world

Electromechanical (EM) Oscillations



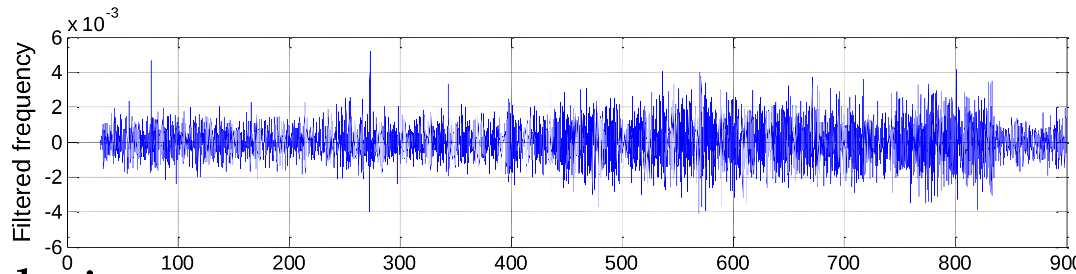
- Can we *infer* the grid *frequency response* to any disturbance input using the ambient synchrophasor data?

A Data-Driven Approach

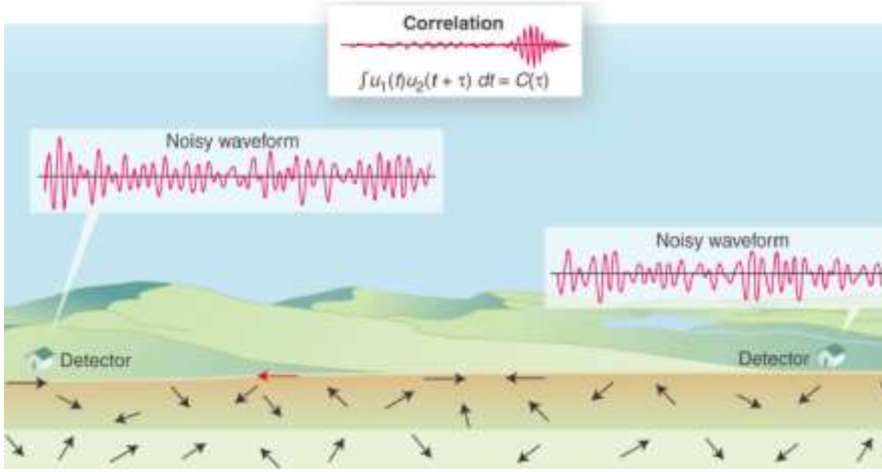


Sample ambient data

Filtered output: white noise!



- In seismology, cross-correlating ambient noise fields successfully used to recover the propagation of earthquake waves [Sneider'04, Wapenaar'04, Sneider et al'07]
  - Analytical results established for *homogeneous continuum medium*



Power System Dynamic Analysis

- Mode estimation of frequency and damping from the correlation of ambient frequency/angle/voltage data
  - Recursive estimation algorithms [Zhou et al'05]
  - Pencil matrix method [Borden et al'13]
  - Fast subspace-based algorithms [Ning et al'15]
- Data-driven estimation of dynamic system model such as the dynamic state Jacobian matrix [Wang et al'16-17]
- Green's function connected to power systems [Backhaus et al'12]
  - Continuum modeling of 2-dimensional EM wave propagation with *homogeneously* placed gens/loads/lines [Parashar et al'04]

Our focus: explore the *analytical* conditions/*practical* limitations of cross-correlation based modeling of (primary) freq. response

Dynamic System Modeling

- Consider a system of n generators with the classical model

$$M_i\ddot{\delta}_i + D_i\dot{\delta}_i = u_i - \sum_{j\in\mathcal{N}_i} P_{ij}$$

  - δ_i ($\omega_i = \dot{\delta}_i$): rotor angle (speed) deviation
  - M_i (D_i): angular momentum (damping coefficient)
  - u_i: local input of power imbalance
  - P_{ij} : power flow from generator i to j (for equivalent network)

- Using the linearized power flow model

$$\mathbf{M}\ddot{\boldsymbol{\delta}} + \mathbf{D}\dot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} = \mathbf{u} \qquad \textbf{(SE)}$$

  - **M** and **D** are diagonal
  - **K** is the power flow Jacobian matrix (~symmetric)

Ambient Data Modeling

- **Goal**: estimating (impulse) frequency response from any u_k to ω_ℓ

$$T_{k\ell}(t) := \omega_\ell(t)\big|_{\mathbf{u}=\delta(t)\mathbf{e}_k}$$

- Ambient conditions: normal operations with small perturbations
  - Random variations of power system loads/resources

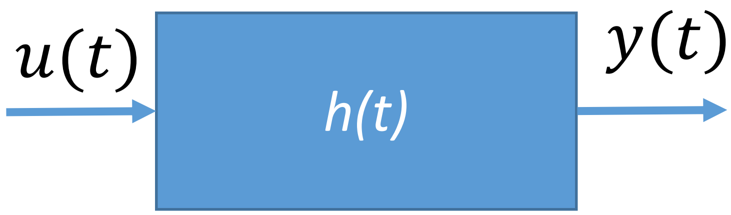
(as1) The system (**SE**) is excited by zero-mean white noise input

$$\mathbb{E}\big[\mathbf{u}(t)\big] = \mathbf{0},$$
$$\mathbb{E}\big[\mathbf{u}(t)\mathbf{u}^{\mathsf{T}}(t-\tau)\big] = \mathbf{\Sigma}\delta(\tau)$$

- (Normalized) cross-correlation of ambient speed (frequency) data

$$C_{k\ell}(\tau) := \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \omega_k(t)\omega_\ell(t-\tau)d\tau$$
$$= \mathbb{E}\big[\omega_k(t)\omega_\ell(t-\tau)\big]$$

A Classical Example



- System identification of SISO
- If input $u(t)$ is white noise, then the transfer function

$$h(t) \propto C_{yu}(t)$$

- Even if $u(t)$ is non-white, can estimate it using $C_{uu}(t)$

Model-based Analysis

- For simplicity, consider *undamped* oscillations with $\mathbf{D} = \mathbf{0}$

$$\mathbf{M}\ddot{\delta} + \mathbf{D}\dot{\delta} + \mathbf{K}\delta = \mathbf{M}\dot{\omega} + \mathbf{K}\delta = \mathbf{u} \qquad \textbf{(SE')}$$

  - Extended to uniformly damped systems (homogeneity relaxed!)

- Oscillation modes for (**SE'**) solved by ***generalized eigen***. problem

$$\mathbf{KC} = \mathbf{MC\Lambda}$$

(as2) $\mathbf{M}$ is positive definite (PD) and $\mathbf{K}$ is symmetric

Lemma: Under ***(as2)***, the eigenvectors in $\mathbf{C}$ are **M**-orthonormal; i.e.,

$$\mathbf{C}^{\mathsf{T}}\mathbf{MC} = \mathbf{I}$$

with $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_n\}$ having eigenvalues of $\mathbf{M}^{-1/2}\mathbf{K}\mathbf{M}^{-1/2}$

Uncoupled Modes

- Linear transformation of (SE′): $\boldsymbol{\delta} = \mathbf{C}\mathbf{z}$ and $\mathbf{v} := \mathbf{C}^{\mathsf{T}}\mathbf{u}$

$$\ddot{\mathbf{z}} = -\boldsymbol{\Lambda}\mathbf{z} + \mathbf{v}$$

- Each mode ($\ddot{z}_i = -\lambda_i z_i + v_i$) associated with $\sqrt{-\lambda_i} = \pm \mathrm{j}\beta_i$
- Under zero initialization

$$\dot{z}_i(t) = \int_0^T \cos(\beta_i \tau)\mathbf{c}_i^{\mathsf{T}}\mathbf{u}(t-\tau)d\tau$$

$$\omega_\ell(t) = \sum_{i=1}^n c_{\ell i}\dot{z}_i(t) = \int_0^T \left[\sum_{i=1}^n c_{\ell i}\cos(\beta_i\tau)\right]\mathbf{c}_i^{\mathsf{T}}\mathbf{u}(t-\tau)d\tau$$

- Impulse frequency response

$$T_{k\ell}(\tau) = \sum_{i=1}^n c_{ki}c_{\ell i}\cos(\beta_i\tau)$$

Equivalence Results

$$\omega_\ell(t) = \int_0^T \left[\sum_{i=1}^{n} c_{\ell i} \cos(\beta_i \tau)\right] \mathbf{v}(t-\tau)d\tau$$

(***as3)*** Input noise variance proportional to inertia; i.e., $\mathbf{\Sigma} = \mu\mathbf{M}$

- Homogeneously excited modes: identical and uncorrelated

$$\mathbb{E}\left[\mathbf{v}(t)\mathbf{v}^{\mathrm{T}}(t-\tau)\right] = \mathbf{C}^{\mathrm{T}}\mathbf{\Sigma}\mathbf{C}\delta(\tau) = \mu\mathbf{I}\delta(\tau)$$

Prop: Under **(*as1*)-(*as3*)**, frequency response can be recovered by cross-correlating ω_k and ω_ℓ as $C_{k\ell}(t) \cong \frac{\mu}{2}T_{k\ell}(t)$

$$C_{k\ell}(\tau) = \mathbb{E}\left[\omega_k(t)\omega_\ell(t-\tau)\right]$$

$$\cong \sum_{i=1}^{n} \mu c_{ki} c_{\ell i}\left[\frac{1}{2}\cos(\beta_i\tau) + \frac{1}{2T}\int_0^T \cos(2\beta_i\tau_1 - \beta_i\tau)d\tau_1\right]$$

Under (as3), only intra-mode components exist!

Damped System Extension

- Under uniform damping, **M**-orthonormal property still holds

$$\mathbf{C}^\top \mathbf{M} \mathbf{C} = \mathbf{I}$$

- Each mode ($\ddot{z}_i + \gamma \dot{z}_i + \lambda_i \dot{z}_i = \mathbf{c}_i^\top \mathbf{u}$) is updated to

$$\dot{z}_i(\tau) = \int_0^\infty \left(a_i e^{c_i t} + b_i e^{d_i t}\right) \mathbf{c}_i^\top \mathbf{u}(\tau - t) dt$$

with

$$a_i = \frac{2\lambda_i}{\sqrt{\gamma^2 - 4\lambda_i}(-\gamma - \sqrt{\gamma^2 - 4\lambda_i})},$$

$$b_i = \frac{-2\lambda_i}{\sqrt{\gamma^2 - 4\lambda_i}(-\gamma + \sqrt{\gamma^2 - 4\lambda_i})},$$

$$c_i = \frac{-\gamma + \sqrt{\gamma^2 - 4\lambda_i}}{2},$$

$$d_i = \frac{-\gamma - \sqrt{\gamma^2 - 4\lambda_i}}{2}.$$

P. Huynh, Q. Chen, A. Elbanna, and H. Zhu, "Data-Driven Estimation of Frequency Response from Ambient Synchrophasor Measurements," *IEEE Trans. Power Systems*, Nov. 2018.

WSCC 9-Bus Test Case

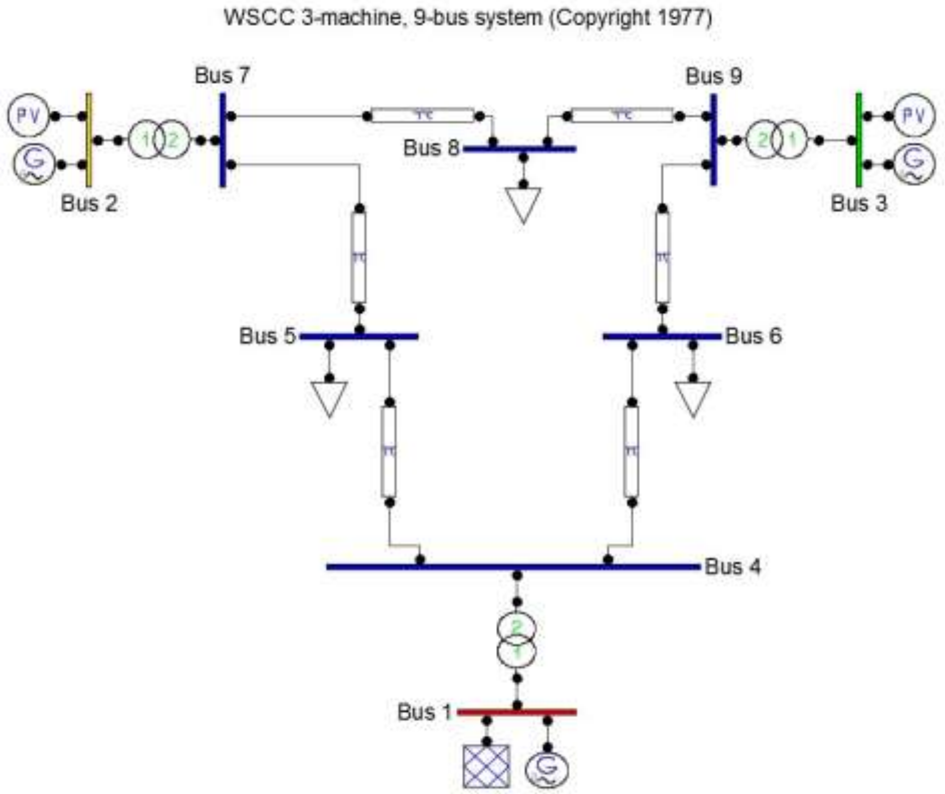
- Synthetic ambient speed outputs generated with randomly perturbing generator inputs using:

  (i) linearized system model

  (ii) time-domain simulation

- With line losses, matrix **K** slightly asymmetric

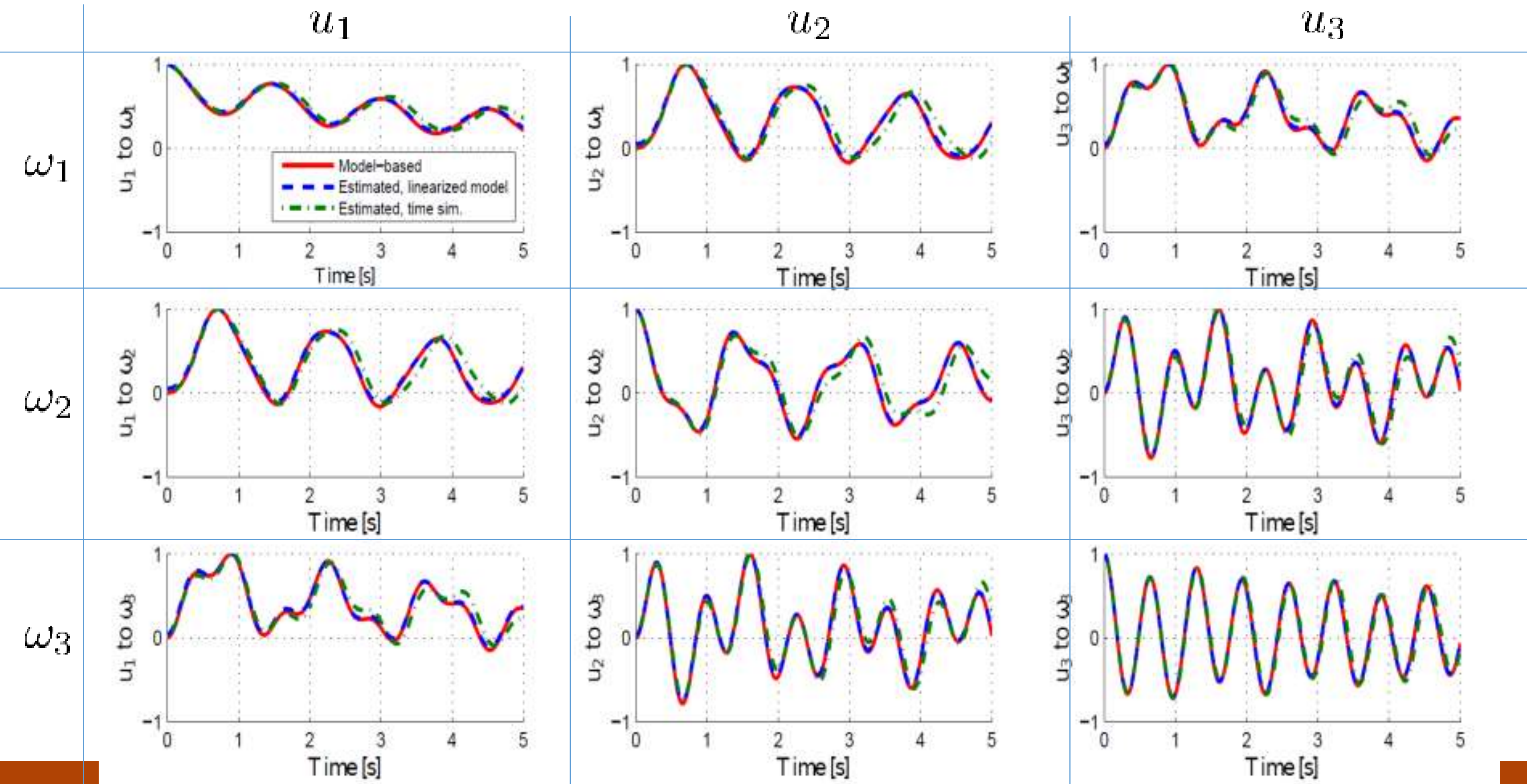
$$\mathbf{K} = \begin{bmatrix} 2.819 & -1.523 & -1.294 \\ -1.611 & 2.723 & -1.112 \\ -1.338 & -1.108 & 2.447 \end{bmatrix}$$



WSCC 3-gen 9-bus case one-line diagram [PSAT]

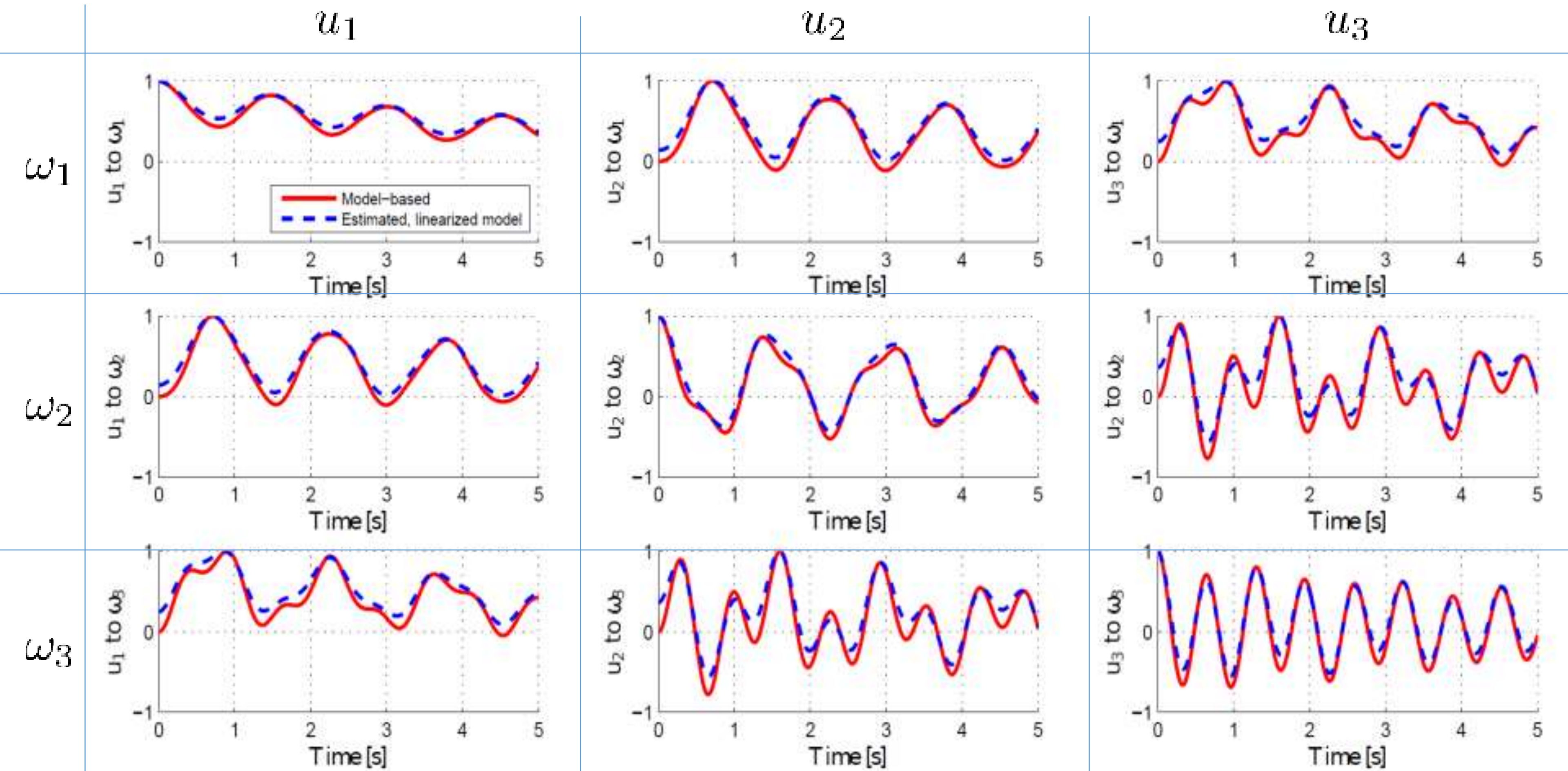
Uniform Damping

- Great match with non-symmetric **K** under line losses!



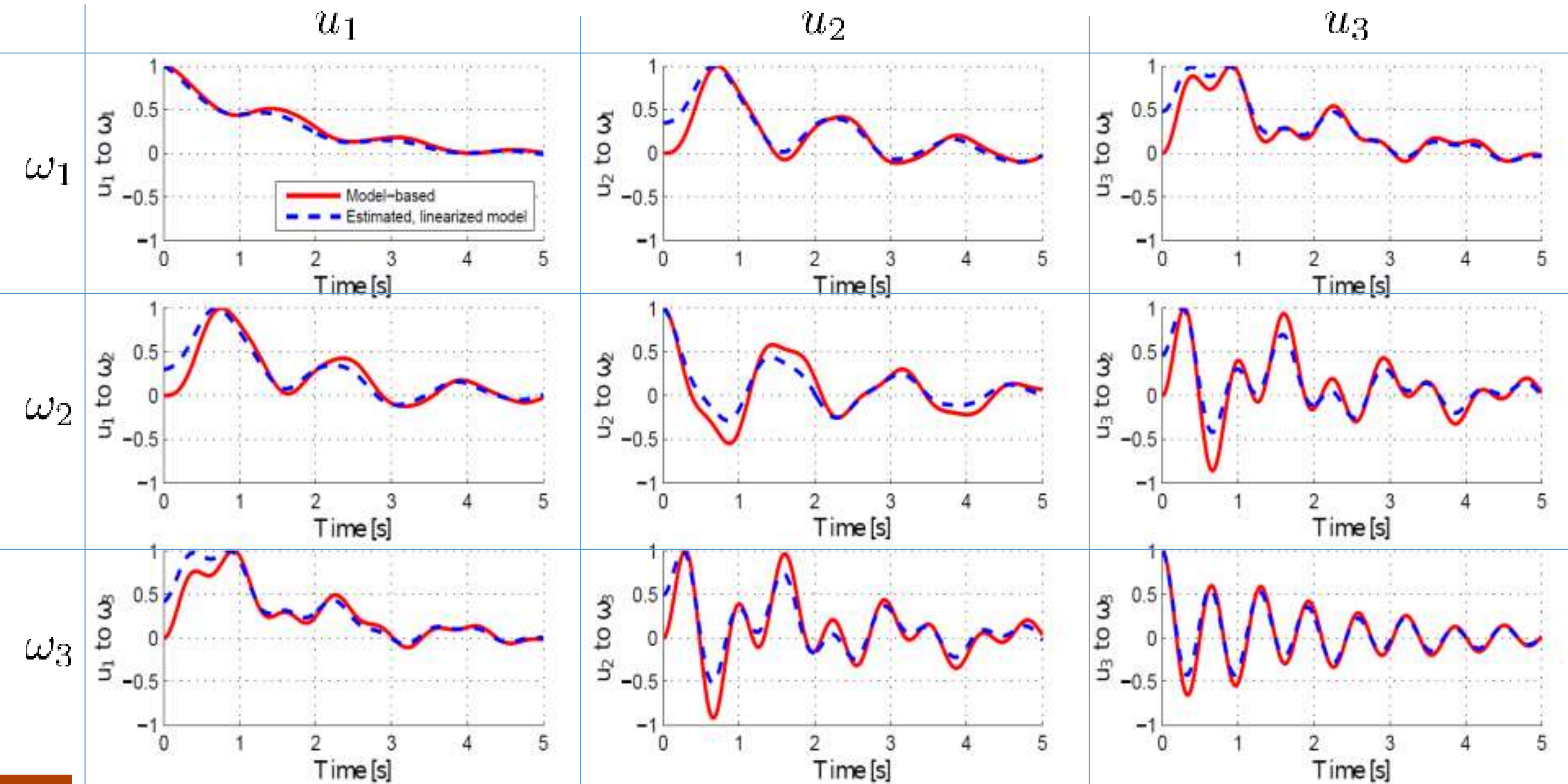
Non-uniform Damping

- Less accurate estimation of scaling factor (mode coupling)



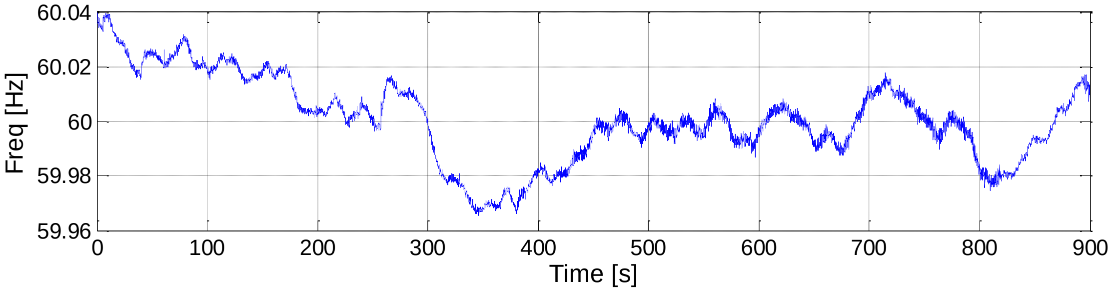
Higher-order Generator Model

- Noticeable difference in the curve shape (correlated modes)



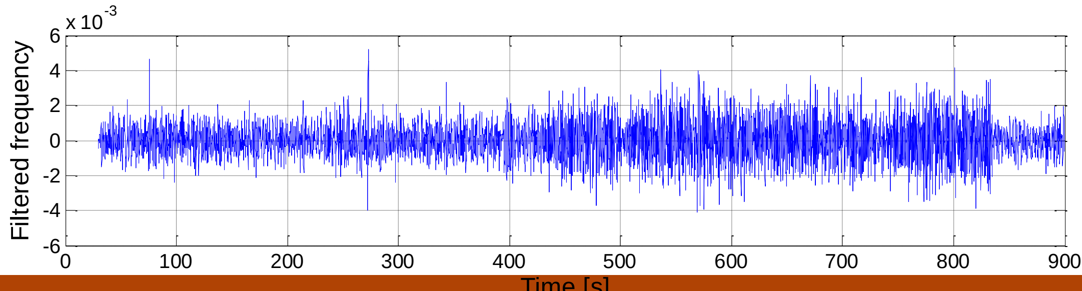
Real Data Tests

- Frequency measurements for the Eastern Interconnection (EI) system under normal grid operations
  - Collected from 10:00-10:15 AM on 01/20/2017 by FNET devices
- Compared to the actual response to the disturbance of 2008 Florida blackout



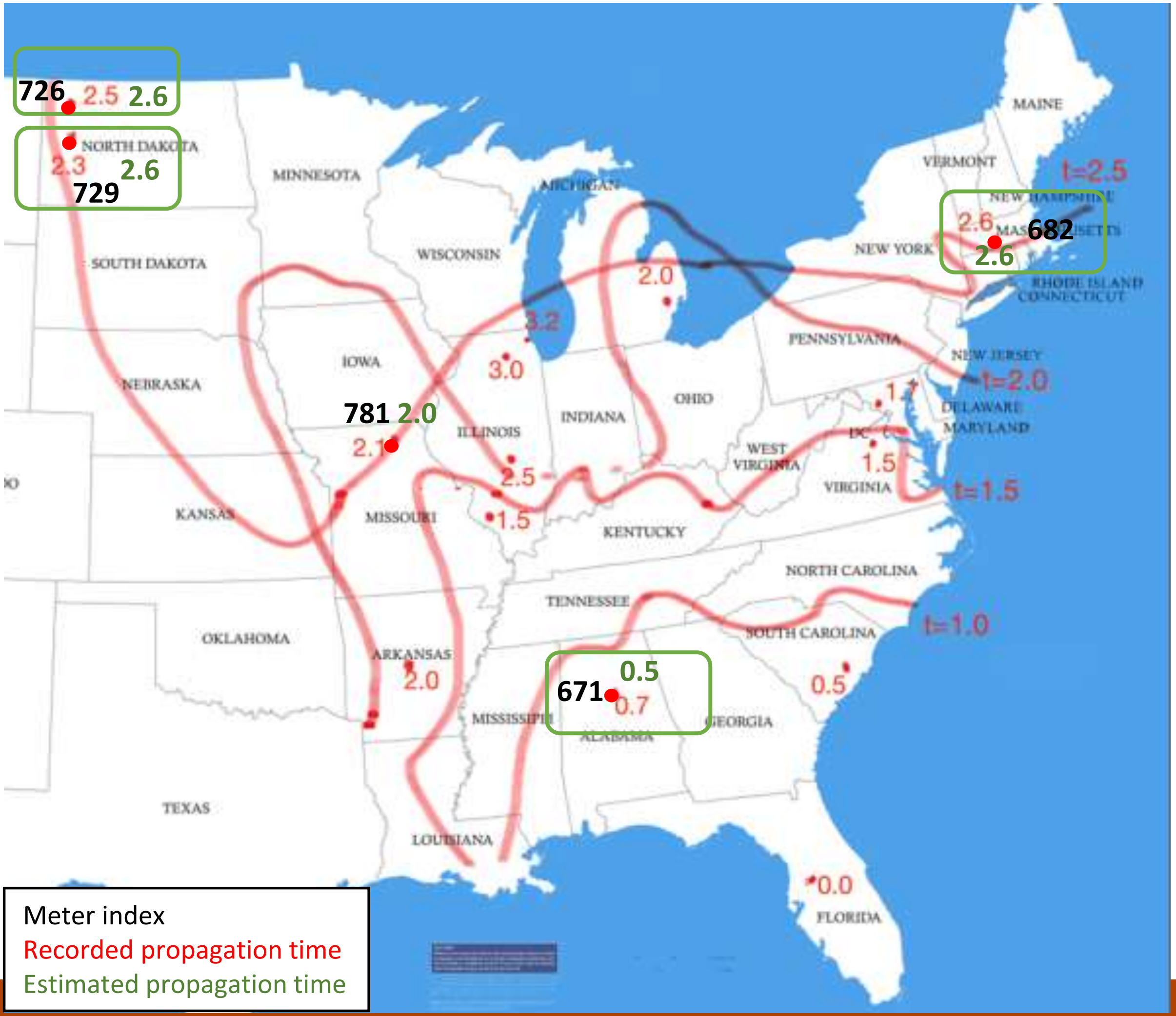
Sample data

Filtered output



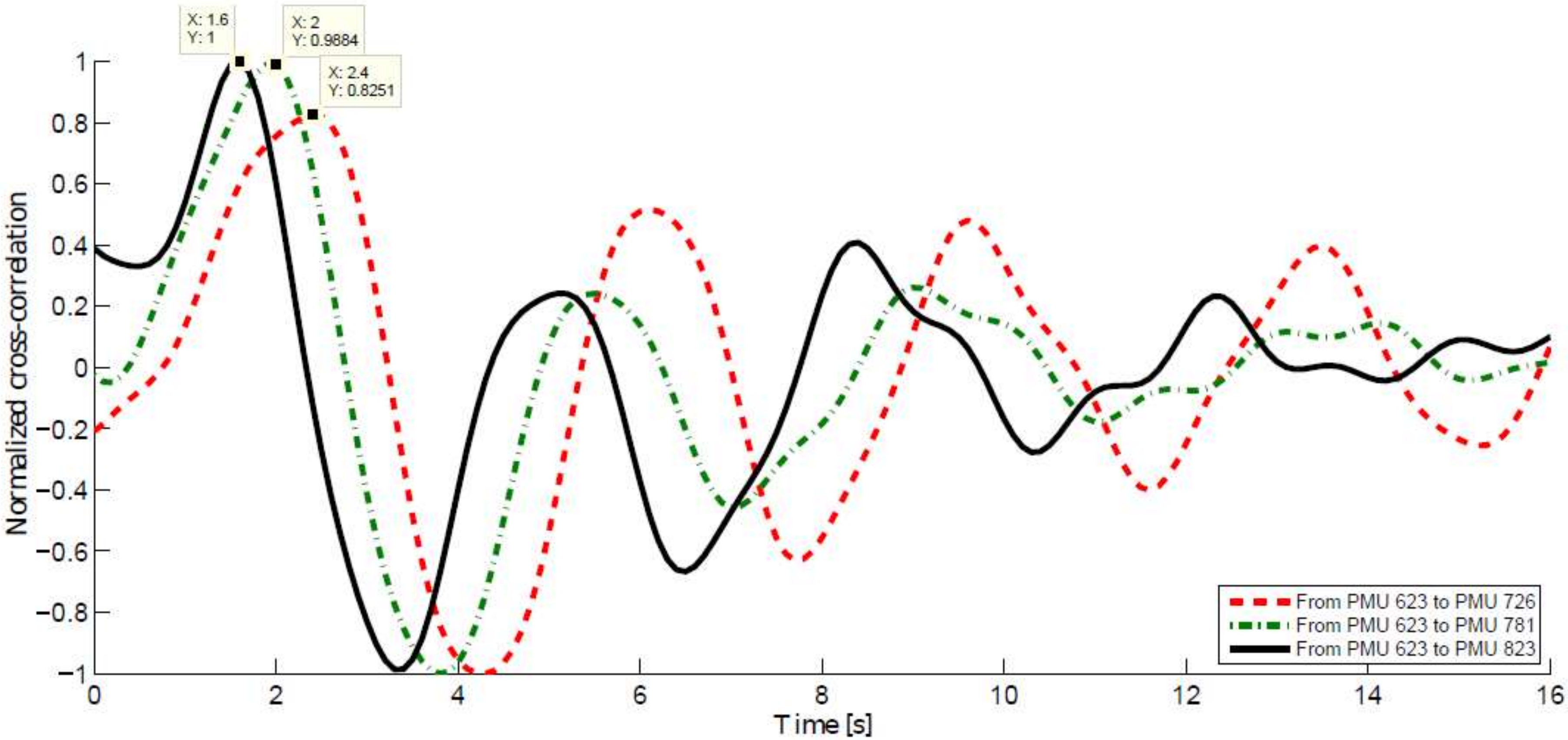
Propagation Time

Node #	Rec.	Est.
601	1.5	1.2
671	0.7	0.5
682	2.6	2.6
726	2.5	2.6
729	2.3	2.6
756	1.6	1.9
767	1.5	1.9
781	2.1	2.1
787	1.6	1.6
823	1.5	1.7



Estimated Response

- From Florida to Arkansas, Missouri, and North Dakota



Conclusions and ?

- Identified a set of analytical conditions to allow the recovery of frequency response using ambient data cross-correlation
  - Uniformly damped system with *uncoupled* modes
  - Each mode *equally excited* by zero-mean perturbations
- These conditions may hold in practice, however, limiting this approach because of the following open questions
  - Account for system nonlinearity
  - Towards high-dimensional space
  - How about *real-time decision making*?

Thank you!

Hao Zhu

haozhu@utexas.edu

http://sites.utexas.edu/haozhu/